Unit 1 – Representing Relationships Mathematically

(3 weeks, August/September)

Common Core State Standards Addressed:

N-Q.1: Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
N-Q.2: Define appropriate quantities for the purpose of descriptive modeling.
A-SSE.1a: Interpret parts of an expression, such as terms, factors, and coefficients.
A-CED.1: Create equations and inequalities in one variable and use them to solve problems.
A-CED.2: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
A-CED.3: Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.
A-REI.10: Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
F-IF.5: Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*[*]
F-IF.9: Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum*.
F-BF.1a: Write a function that describes a relationship between two quantities.[*]

Student Friendly Learning Targets (In order of teaching):

- I can use unit analysis and perform unit conversions.
- I can identify parts of an expression.
- I can write the equation or inequality that best models the problem.
- I can interpret solutions in the context of the situation modeled and decide if they are reasonable.
- I can verify that any point on a graph will result in a true equation when their coordinates are substituted into the equation.
- I can state the appropriate domain of a function that represents a problem situation, defend my choice, and explain why other numbers might be excluded from the domain.
- I can compare properties of two functions graphically, in table form, and algebraically.

Vocabulary:

expression, equation, term, variable, coefficient, constant, like terms, degree, inequality, viable, non-viable, ordered pair, coordinate plane, axes, coordinate, function, domain, range, x-intercept, y-intercept

Materials and/or Technology Needed:
calculator, graphing calculator, Desmos.com, graph paper, ruler

Instructional Strategies:
Information from all Learning Targets can be found at Ohio's Model Curricula: http://education.ohio.gov/Topics/Academic-Content-Standards/Mathematics and KCTM's Flip Book (which encompasses several states' information including Ohio): http://katm.org/wp/wp-content/uploads/flipbooks/High-School-CCSS-Flip-Book-USD-259-2012.pdf

- **I can use unit analysis and perform unit conversions.**

(From Ohio's Model Curricula) Students need to develop sound mathematical reasoning skills and forms of argument to make reasonable judgments about their solutions. They should be able to decide whether a problem calls for an estimate, for an approximation, or for an exact answer. To accomplish this goal, teachers should provide students with a broad range of contextual problems that offer opportunities for performing operations with quantities involving units. These problems should be connected to science, engineering, economics, finance, medicine, etc.

Some contextual problems may require an understanding of derived measurements and capability in unit analysis. Keeping track of derived units during computations and making reasonable estimates and rational conclusions about accuracy and the precision of the answers help in the problem-solving process.

The computation shows that the gasoline is less expensive in the United States and how an analysis can be helpful in keeping track of unit conversations. Students should be able to correctly identify the degree of precision of the answers which should not be far greater than the actual accuracy of the measurements.

(From the KCTM Flip Book) Interpret units in the context of the problem. When solving a multi-step problem, use units to evaluate the appropriateness of the solution. Choose the appropriate units for a specific formula and interpret the meaning of the unit in that context. Choose and interpret both the scale and the origin in graphs and data displays.

Include word problems where quantities are given in different units, which must be converted to make sense of the problem.

Common Misconceptions (from Ohio's Model Curricula):
Students may not realize the importance of the units' conversions in conjunction with the computation when solving problems involving measurements.

Since today's calculating devices often display 8 to 10 decimal places, students frequently express answers to a much greater degree of precision than the required.

- **I can identify parts of an expression.**

(From KCTM Flip Book) Extending beyond simplifying an expression, this cluster addresses interpretation of the components in an algebraic expression. A student should recognize that in the expression 2x + 1, "2" is the coefficient, "2" and "x" are factors, and "1" is a constant, as well as "2x" and "1" being terms of the binomial expression. Development and proper use of mathematical language is an important building block for future content. Using real-world context examples, the nature of algebraic expressions can be explored.

Common Misconceptions (from KCTM Flip Book):
Students may believe that the use of algebraic expressions is merely the abstract manipulation of symbols. Use of real- world context examples to demonstrate the meaning of the parts of algebraic expressions is needed to counter this misconception.

Students will often combine terms that are not like terms. For example, 2 + 3x = 5x or 3x + 2y = 5xy.

Students sometimes forget the coefficient of 1 when adding like terms. For example, x + 2x + 3x = 5x rather than 6x.

Students will change the degree of the variable when adding/subtracting like terms. For example, $2x + 3x = 5x^2$ rather than 5x.

Students will forget to distribute to all terms when multiplying. For example, 6(2x + 1) = 12x + 1 rather than 12x + 6.

Students may not follow the Order of Operations when simplifying expressions. For example, $4x^2$ when x = 3 may be incorrectly evaluated as $4•32 = 12^2 = 144$, rather than 4•9 = 36. Another common mistake occurs when the distributive property should be used prior to adding/subtracting. For example, 2 + 3(x – 1) incorrectly becomes 5(x – 1) = 5x – 5 instead of 2 + 3(x – 1) = 2 + 3x – 3 = 3x – 1.

Students fail to use the property of exponents correctly when using the distributive property. For example, 3x(2x – 1) = 6x – 3x = 3x instead of simplifying as $3x(2x – 1) = 6x^2 – 3x$.

Students fail to understand the structure of expressions. For example, they will write 4x when x = 3 is 43 instead of 4x = 4•x so when x = 3, 4x = 4•3 = 12. In addition, students commonly misevaluate $–3^2 = 9$ rather than $–3^2 = –9$. Students routinely see $–3^2$ as the same as $(–3)^2 = 9$. A method that may clear up the misconception is to have students rewrite as $–x^2 = –1•x^2$ so they know to apply the exponent before the multiplication of –1.

Students frequently attempt to "solve" expressions. Many students add "= 0" to an expression they are asked to simplify. Students need to understand the difference between an equation and an expression.

Students commonly confuse the properties of exponents, specifically the product of powers property with the power of a power property. For example, students will often simplify $(x^2)^3 = x^5$ instead of x^6.

Students will incorrectly translate expressions that contain a difference of terms. For example, 8 less than 5 times a number is often incorrectly translated as 8 – 5n rather than 5n – 8.

- **I can write the equation or inequality that best models the problem.**

(From Ohio's Model Curricula) Using real-world context examples, the nature of algebraic expressions can be explored**.** For example, suppose the cost of cell phone service for a month is represented by the expression $0.40s + 12.95$**.** Students can analyze how the coefficient of 0.40 represents the cost of one minute (40¢), while the constant of 12.95 represents a fixed, monthly fee, and *s* stands for the number of cell phone minutes used in the month. Similar real-world examples, such as tax rates, can also be used to explore the meaning of expressions.

Provide examples of real-world problems that can be modeled by writing an equation or inequality. Begin with simple equations and inequalities and build up to more complex equations in two or more variables that may involve quadratic, exponential or rational functions.

Explore examples illustrating when it is useful to rewrite a formula by solving for one of the variables in the formula. For example, the formula for the area of a trapezoid ($A = \frac{1}{2}h(b_1 + b_2)$) can be solved for *h* if the area and lengths of the bases are known but the height needs to be calculated. This strategy of selecting a different representation has many applications in science and business when using formulas.

Provide examples of real-world problems that can be solved by writing an equation, and have students explore the graphs of the equations on a graphing calculator to determine which parts of the graph are relevant to the problem context.

Give students formulas, such as area and volume (or from science or business), and have students solve the equations for each of the different variables in the formula.

Provide a real-world example (e.g., a table showing how far a car has driven after a given number of minutes, traveling at a uniform speed), and examine the table by looking "down" the table to describe a recursive relationship, as well as "across" the table to determine an explicit formula to find the distance traveled if the number of minutes is known.

Write out terms in a table in an expanded form to help students see what is happening. For example, if the *y*-values are 2, 4, 8, 16, they could be written as 2, 2(2), 2(2)(2), 2(2)(2)(2), etc., so that students recognize that 2 is being used multiple times as a factor.

Focus on one representation and its related language – recursive or explicit – at a time so that students are not confusing the formats.

<u>Common Misconceptions (from Ohio's Model Curricula):</u>
Students may believe that equations of linear, quadratic and other functions are abstract and exist only "in a math book," without seeing the usefulness of these functions as modeling real-world phenomena.

(From KCTM Flip Book) Students may interchange slope and y-intercept when creating equations. For example, a taxi cab costs $4 for a dropped flag and charges $2 per mile. Students may fail to see that $2 is a rate of change and is slope while the $4 is the starting cost and incorrectly write the equation as $y = 4x + 2$ instead of $y = 2x + 4$.

Students do not correctly identify whether a situation should be represented by a linear, quadratic, or exponential function.

Students often do not understand what the variables represent. For example, if the height h in feet of a piece of lava t seconds after it is ejected from a volcano is given by $h(t) = -16t^2 + 64t + 936$ and the student is asked to find the time it takes for the piece of lava to hit the ground, the student will have difficulties understanding that $h = 0$ at the ground and that they need to solve for t.

- **I can interpret solutions in the context of the situation modeled and decide if they are reasonable.**

(From Ohio's Model Curricula) Examine real-world graphs in terms of constraints that are necessary to balance a mathematical model with the real-world context. For example, a student writing an equation to model the maximum area when the perimeter of a rectangle is 12 inches should recognize that $y = x(6 – x)$ only makes sense when $0 < x < 6$. This restriction on the domain is necessary because the side of a rectangle under these conditions cannot be less than or equal to 0, but must be less than 6. Students can discuss the difference between the parabola that models the problem and the portion of the parabola that applies to the context.

<u>Common Misconceptions (from Ohio's Model Curricula):</u>
Students may believe that equations of linear, quadratic and other functions are abstract and exist only "in a math book," without seeing the usefulness of these functions as modeling real-world phenomena.

- **I can verify that any point on a graph will result in a true equation when their coordinates are substituted into the equation.**

(From Ohio's Model Curricula) Discuss the importance of using appropriate labels and scales on the axes when representing functions with graphs.

Use a graphing calculator or Desmos.com to demonstrate how dramatically the shape of a curve can change when the scale of the graph is altered for one or both variables.

Beginning with simple, real-world examples, help students to recognize a graph as a set of solutions to an equation. For example, if the equation $y = 6x + 5$ represents the amount of money paid to a babysitter (i.e., $5 for gas to drive to the job and $6/hour to do the work), then every point on the line represents an amount of money paid, given the amount of time worked.

Using technology, have students graph a function and use the trace function to move the cursor along the curve. Discuss the meaning of the ordered pairs that appear at the bottom of the calculator, emphasizing that every point on the curve represents a solution to the equation. Begin with simple linear equations and how to solve them using the graphs and tables on a graphing calculator. Then, advance students to nonlinear situations so they can see that even complex equations that might involve quadratics, absolute value, or rational functions can be solved fairly easily using this same strategy. While a standard graphing calculator does not simply solve an equation for the user, it can be used as a tool to approximate solutions.

Common Misconceptions (from Ohio's Model Curricula):
Students believe that the labels and scales on a graph are not important and can be assumed by a reader, and that it is always necessary to use the entire graph of a function when solving a problem that uses that function as its model.

Students may believe that the graph of a function is simply a line or curve "connecting the dots," without recognizing that the graph represents all solutions to the equation.

Additionally, students may believe that two-variable inequalities have no application in the real world.

- **I can state the appropriate domain of a function that represents a problem situation, defend my choice, and explain why other numbers might be excluded from the domain.**

(From Ohio's Model Curricula) Recognize appropriate domains of functions in real-world settings. For example, when determining a weekly salary based on hours worked, the hours (input) could be a rational number, such as 25.5. However, if a function relates the number of cans of soda sold in a machine to the money generated, the domain must consist of whole numbers

Common Misconceptions (from Ohio's Model Curricula):
Students may believe that it is reasonable to input any x-value into a function, so they will need to examine multiple situations in which there are various limitations to the domains.

- **I can compare properties of two functions graphically, in table form, and algebraically.**

(From Ohio's Model Curricula) Graphical representations serve as visual models for understanding phenomena that take place in our daily surroundings. The use of different kinds of graphical representations along with their units, labels and titles demonstrate the level of students' understanding and foster the ability to reason, prove, self-check, examine relationships and establish the validity of arguments. Students need to be able to identify misleading graphs by choosing correct units and scales to create a correct representation of a situation or to make a correct conclusion from it.

Discuss the importance of using appropriate labels and scales on the axes when representing functions with graphs.

Use a graphing calculator or Desmos.com to demonstrate how dramatically the shape of a curve can change when the scale of the graph is altered for one or both variables.

Common Misconceptions (from Ohio's Model Curricula):
Students believe that the labels and scales on a graph are not important and can be assumed by a reader, and that it is always necessary to use the entire graph of a function when solving a problem that uses that function as its model.

(From KCTM Flip Book) Given a graph of a line, students use the x-intercept for b instead of the y-intercept.

Given a graph, students incorrectly compute slope as run over rise rather than rise over run. For example, they will compute slope with the change in x over the change in y.

Strategies for Diverse Learners:
Problems of varying difficulty are incorporated into the problem sets.

Literacy Standards Considerations:

- **I can use unit analysis and perform unit conversions.**

Students would need to interpret the situation to determine which units may be needed to change to the answer required.

- **I can identify parts of an expression.**

Students would need to be familiar with the definitions of key terms.

- **I can write the equation or inequality that best models the problem.**

Word Problems are included as a part of the practice set - students have to interpret into algebraic notation.

- **I can interpret solutions in the context of the situation modeled and decide if they are reasonable.**

Word Problems are included as a part of the practice set - students have to interpret into algebraic notation.

- **I can verify that any point on a graph will result in a true equation when their coordinates are substituted into the equation.**

Students will need to interpret the problem situation to determine what answers make sense.

- **I can state the appropriate domain of a function that represents a problem situation, defend my choice, and explain why other numbers might be excluded from the domain.**

Students will need to interpret the problem situation to determine what answers make sense.

- **I can compare properties of two functions graphically, in table form, and algebraically.**

Students will need to interpret the problem situation to determine what answers make sense.

Instructional Resources:
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- **I can use unit analysis and perform unit conversions.**

http://www2.franciscan.edu/academic/mathsci/mathscienceintegation/MathScienceIntegation-620.htm
http://serc.carleton.edu/mathyouneed/units/UnitExample.html
http://spacemath.gsfc.nasa.gov/weekly/6Page53.pdf
http://everybodyisageniusblog.blogspot.com/2013/01/doing-less.html
http://ispeakmath.org/2013/08/03/turbo-fun-conversion-percent-problem/
http://mathforum.org/pows/viewPublication.htm?id=4340 (requires subscription)
http://www.achieve.org/math-works-brochures (has several different occupations – free PDFs)

- **I can identify parts of an expression.**

http://map.mathshell.org/materials/lessons.php?taskid=221&subpage=concept

- **I can write the equation or inequality that best models the problem.**

http://everybodyisageniusblog.blogspot.com/2012/09/translating-math.html
http://www.insidemathematics.org/problems-of-the-month/pom-growingstaircases.pdf

- **I can interpret solutions in the context of the situation modeled and decide if they are reasonable.**

http://www.insidemathematics.org/common-core-math-tasks/high-school/HS-A-2003%20Number%20Towers.pdf
http://www.insidemathematics.org/problems-of-the-month/pom-surroundedandcovered.pdf
http://www.insidemathematics.org/problems-of-the-month/pom-perfectpair.pdf
Many of the resources for writing the equation or inequality that best models the problem also will work here.

- **I can verify that any point on a graph will result in a true equation when their coordinates are substituted into the equation.**

http://www.insidemathematics.org/common-core-math-tasks/high-school/HS-A-2006%20Graphs2006.pdf

- **I can state the appropriate domain of a function that represents a problem situation, defend my choice, and explain why other numbers might be excluded from the domain.**

http://www.illustrativemathematics.org/illustrations/387

- **I can compare properties of two functions graphically, in table form, and algebraically.**

http://www.insidemathematics.org/common-core-math-tasks/high-school/HS-F-2007%20Graphs2007.pdf

Assessment:

Formative:

- exit slips
- observation of students working on problems in class
- observation of student work from outside of class practice problems

Summative:

- unit test questions

Assessment Resources:

http://www.illustrativemathematics.org/
http://practice.parcc.testnav.com/
IIS
Holt McDougal Algebra 1 © 2011

Unit 2 – Understanding Functions

(2 ½ weeks, September)

Common Core State Standards Addressed:

F-IF.1: Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.
F-IF.2: Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
F-IF.3: Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by* $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ *for* $n \geq 1$.
F-IF.4: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity*.*
F-IF.5: Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.**

Student Friendly Learning Targets (In order of teaching):

- I can determine if a relation is a function.
- I can evaluate functions using function notation.
- I can recognize that sequences are functions.
- I can find key features of a graph using a graph, a table, or an equation.
- I can relate the domain of a function to its graph or the relationship it describes using real-life problems.

Vocabulary:

function, domain, range, input, output, function notation, arithmetic sequence, maximum, minimum, end behavior increasing, decreasing

Materials and/or Technology Needed:

calculator, graphing calculator, Desmos.com, graph paper, ruler

Instructional Strategies:

Information from all Learning Targets can be found at Ohio's Model Curricula: http://education.ohio.gov/Topics/Academic-Content-Standards/Mathematics and KCTM's Flip Book (which encompasses several states' information including Ohio): http://katm.org/wp/wp-content/uploads/flipbooks/High-School-CCSS-Flip-Book-USD-259-2012.pdf

- **I can determine if a relation is a function.**

(From Ohio's Model Curricula) Provide applied contexts in which to explore functions. For example, examine the amount of money earned when given the number of hours worked on a job, and contrast this with a situation in which a single fee is paid by the "carload" of people, regardless of whether 1, 2, or more people are in the car.

Help students to understand that the word "domain" implies the set of all possible input values and that the integers are a set of numbers made up of {…-2, -1, 0, 1, 2, …}.

Distinguish between relationships that are not functions and those that are functions (e.g., present a table in which one of the input values results in multiple outputs to contrast with a functional relationship). Examine graphs of functions and non-functions, recognizing that if a vertical line passes through at least two points in the graph, then y (or the quantity on the vertical axis) is not a function of x (or the quantity on the horizontal axis).

<u>Common Misconceptions (from Ohio's Model Curricula):</u>
Students may believe that all relationships having an input and an output are functions, and therefore, misuse the function terminology.

- **I can evaluate functions using function notation.**

(From Ohio's Model Curricula) Provide applied contexts in which to explore functions. For example, examine the amount of money earned when given the number of hours worked on a job, and contrast this with a situation in which a single fee is paid by the "carload" of people, regardless of whether 1, 2, or more people are in the car.

Use diagrams to help students visualize the idea of a function machine. Students can examine several pairs of input and output values and try to determine a simple rule for the function.

<u>Common Misconceptions (from Ohio's Model Curricula):</u>
Students may also believe that the notation $f(x)$ means to multiply some value f times another value x. The notation alone can be confusing and needs careful development. For example, $f(2)$ means the output value of the function f when the input value is 2.

- **I can recognize that sequences are functions.**

(From Ohio's Model Curricula) Rewrite sequences of numbers in tabular form, where the input represents the term number (the position or index) in the sequence, and the output represents the number in the sequence.

- **I can find key features of a graph using a graph, a table, or an equation.**

(From Ohio's Model Curricula) Flexibly move from examining a graph and describing its characteristics (e.g., intercepts, relative maximums, etc.) to using a set of given characteristics to sketch the graph of a function.

Examine a table of related quantities and identify features in the table, such as intervals on which the function increases, decreases, or exhibits periodic behavior.

Begin with simple, linear functions to describe features and representations, and then move to more advanced functions, including non-linear situations.

Provide students with many examples of functional relationships, both linear and non-linear. Use real-world examples, such as the growth of an investment fund over time, so that students can not only describe what they see in a table, equation, or graph, but also can relate the features to the real-life meanings.

Allow students to collect their own data sets, such as the falling temperature of a glass of hot water when removed from a flame versus the amount of time, to generate tables and graphs for discussion and interpretation.

- **I can relate the domain of a function to its graph or the relationship it describes using real-life problems.**

(From Ohio's Model Curricula) Recognize appropriate domains of functions in real-world settings. For example, when determining a weekly salary based on hours worked, the hours (input) could be a rational number, such as 25.5. However, if a function relates the number of cans of soda sold in a machine to the money generated, the domain must consist of whole numbers.

Begin with simple, linear functions to describe features and representations, and then move to more advanced functions, including non-linear situations.

Provide students with many examples of functional relationships, both linear and non-linear. Use real-world examples, such as the growth of an investment fund over time, so that students can not only describe what they see in a table, equation, or graph, but also can relate the features to the real-life meanings.

Allow students to collect their own data sets, such as the falling temperature of a glass of hot water when removed from a flame versus the amount of time, to generate tables and graphs for discussion and interpretation.

<u>Common Misconceptions (from Ohio's Model Curricula):</u>
Students may believe that it is reasonable to input any *x*-value into a function, so they will need to examine multiple situations in which there are various limitations to the domains.

<u>Strategies for Diverse Learners:</u>
Problems of varying difficulty are incorporated into the problem sets.

<u>Literacy Standards Considerations:</u>

- **I can determine if a relation is a function.**

Students would need to be familiar with the definitions of key terms.

- **I can evaluate functions using function notation.**

Students would need to be familiar with the definitions of key terms.

- **I can recognize that sequences are functions.**

Students would need to be familiar with the definitions of key terms.

- **I can find key features of a graph using a graph, a table, or an equation.**

Students would need to be familiar with the definitions of key terms.

- **I can relate the domain of a function to its graph or the relationship it describes using real-life problems.**

Students would need to be familiar with the definitions of key terms.

Instructional Resources:
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- **I can determine if a relation is a function.**

http://mathtalesfromthespring.blogspot.com/2012/06/determining-whether-relation-is.html
http://everybodyisageniusblog.blogspot.com/2012/12/teaching-functions.html

- **I can evaluate functions using function notation.**

http://everybodyisageniusblog.blogspot.com/2013/03/function-machines.html

- **I can recognize that sequences are functions.**

http://www.insidemathematics.org/problems-of-the-month/pom-tritriangles.pdf

- **I can find key features of a graph using a graph, a table, or an equation.**

http://squarerootofnegativeoneteachmath.blogspot.com/2013/12/domain-range-from-graph.html

- **I can relate the domain of a function to its graph or the relationship it describes using real-life problems.**

Assessment:
Formative:

- exit slips
- observation of students working on problems in class
- observation of student work from outside of class practice problems

Summative:

- unit test questions

Assessment Resources:

http://www.illustrativemathematics.org/
http://practice.parcc.testnav.com/
IIS
Holt McDougal Algebra 1 © 2011

Unit 3 – Linear Functions

(3 ½ weeks, October)

Common Core State Standards Addressed:

F-IF.6: Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*
F-IF.7a: Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.* Graph linear and quadratic functions and show intercepts, maxima, and minima.
F-BF.1a: Write a function that describes a relationship between two quantities.*
F-LE.1a: Distinguish between situations that can be modeled with linear functions and with exponential functions. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
F-LE.1b: Distinguish between situations that can be modeled with linear functions and with exponential functions. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
F-LE.2: Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
F-LE.5: Interpret the parameters in a linear or exponential function in terms of a context.
S-ID.7: Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

Student Friendly Learning Targets (In order of teaching):

- I can calculate and interpret the average rate of change of a function.
- I can graph a linear function and identify its intercepts.
- I can demonstrate that a linear function has a constant rate of change.
- I can identify situations in which one quantity changes at a constant rate.
- I can construct linear functions from an arithmetic sequence, graph, table of values, or description of the relationship.
- I can interpret the parameters of a linear function in a real-life problem.

Vocabulary:

function, rate of change, average rate of change, interval, slope, evaluate, domain, range, input, output, equation, x-intercept, y-intercept, linear function, coordinate plane, linear function, arithmetic sequence, linear equation, linear model, units, data

Materials and/or Technology Needed:

calculator, graphing calculator, Desmos.com, graph paper, ruler

Instructional Strategies:

Information from all Learning Targets can be found at Ohio's Model Curricula: http://education.ohio.gov/Topics/Academic-Content-Standards/Mathematics and KCTM's Flip

Book (which encompasses several states' information including Ohio): http://katm.org/wp/wp-content/uploads/flipbooks/High-School-CCSS-Flip-Book-USD-259-2012.pdf

- **I can calculate and interpret the average rate of change of a function.**

(From Ohio's Model Curricula) Given a table of values, such as the height of a plant over time, students can estimate the rate of plant growth. Also, if the relationship between time and height is expressed as a linear equation, students should explain the meaning of the slope of the line. Finally, if the relationship is illustrated as a linear or non-linear graph, the student should select points on the graph and use them to estimate the growth rate over a given interval.

The key is that two quantitative variables are being measured on the same subject. The paired data should be listed and then displayed in a scatterplot. If time is one of the variables, it usually goes on the horizontal axis. That which is being predicted goes on the vertical; the predictor variable is on the horizontal axis.

Note that unlike a two-dimensional graph in mathematics, the scales of a scatterplot need not be the same, and even if they are similar (such as SAT Math and SAT Verbal), they still need not have the same spacing. So, visual rendering of slope makes no sense in most scatterplots, i.e., a 45 degree line on a scatterplot need not mean a slope of 1.

Often the interpretation of the intercept (constant term) is not meaningful in the context of the data. For example, this is the case when the zero point on the horizontal is of considerable distance from the values of the horizontal variable, or in some case has no meaning such as for SAT variables.

<u>Common Misconceptions (from Ohio's Model Curricula):</u>
Students may also believe that the slope of a linear function is merely a number used to sketch the graph of the line. In reality, slopes have real-world meaning, and the idea of a rate of change is fundamental to understanding major concepts from geometry to calculus.

Students may believe that a 45 degree line in the scatterplot of two numerical variables always indicates a slope of 1 which is the case only when the two variables have the same scaling. Because the scaling for many real-world situation varies greatly students need to be give opportunity to compare graphs of differing scale. Asking students questions like; What would this graph look like with a different scale or using this scale? Is essential in addressing this misconception.

- **I can graph a linear function and identify its intercepts.**

Use various representations of the same function to emphasize different characteristics of that function.

<u>Common Misconceptions (from Ohio's Model Curricula):</u>

Additionally, students may believe that the process of rewriting equations into various forms is simply an algebra symbol manipulation exercise, rather than serving a purpose of allowing different features of the function to be exhibited.

- **I can demonstrate that a linear function has a constant rate of change.**

(From Ohio's Model Curricula) Compare tabular representations of a variety of functions to show that linear functions have a first common difference (i.e., equal differences over equal intervals), while exponential functions do not (instead function values grow by equal factors over equal *x*-intervals).

Apply linear and exponential functions to real-world situations. For example, a person earning $10 per hour experiences a constant rate of change in salary given the number of hours worked, while the number of bacteria on a dish that doubles every hour will have equal factors over equal intervals.

Provide examples of arithmetic and geometric sequences in graphic, verbal, or tabular forms, and have students generate formulas and equations that describe the patterns.

Use a graphing calculator or computer program to compare tabular and graphic representations of exponential and polynomial functions to show how the *y* (output) values of the exponential function eventually exceed those of polynomial functions.

<u>Common Misconceptions (from Ohio's Model Curricula):</u>
Students may believe that all functions have a first common difference and need to explore to realize that, for example, a quadratic function will have equal second common differences in a table.

- **I can identify situations in which one quantity changes at a constant rate.**

(From Ohio's Model Curricula) Compare tabular representations of a variety of functions to show that linear functions have a first common difference (i.e., equal differences over equal intervals), while exponential functions do not (instead function values grow by equal factors over equal *x*-intervals).

Apply linear and exponential functions to real-world situations. For example, a person earning $10 per hour experiences a constant rate of change in salary given the number of hours worked, while the number of bacteria on a dish that doubles every hour will have equal factors over equal intervals.

Provide examples of arithmetic and geometric sequences in graphic, verbal, or tabular forms, and have students generate formulas and equations that describe the patterns.

Use a graphing calculator or computer program to compare tabular and graphic representations of exponential and polynomial functions to show how the *y* (output) values of the exponential function eventually exceed those of polynomial functions.

Common Misconceptions (from Ohio's Model Curricula):
Students may believe that all functions have a first common difference and need to explore to realize that, for example, a quadratic function will have equal second common differences in a table.

- **I can construct linear functions from an arithmetic sequence, graph, table of values, or description of the relationship.**

(From Ohio's Model Curricula) Provide a real-world example (e.g., a table showing how far a car has driven after a given number of minutes, traveling at a uniform speed), and examine the table by looking "down" the table to describe a recursive relationship, as well as "across" the table to determine an explicit formula to find the distance traveled if the number of minutes is known.

Write out terms in a table in an expanded form to help students see what is happening. For example, if the *y*-values are 2, 4, 8, 16, they could be written as 2, 2(2), 2(2)(2), 2(2)(2)(2), etc., so that students recognize that 2 is being used multiple times as a factor.

Common Misconceptions (from Ohio's Model Curricula):
Students may believe that the best (or only) way to generalize a table of data is by using a recursive formula. Students naturally tend to look "down" a table to find the pattern but need to realize that finding the 100th term requires knowing the 99th term unless an explicit formula is developed.

Students may also believe that arithmetic and geometric sequences are the same. Students need experiences with both types of sequences to be able to recognize the difference and more readily develop formulas to describe them.

- **I can interpret the parameters of a linear function in a real-life problem.**

(From Ohio's Model Curricula) Use real-world contexts to help students understand how the parameters of linear and exponential functions depend on the context. For example, a plumber who charges \$50 for a house call and \$85 per hour would be expressed as the function $y = 85x + 50$, and if the rate were raised to \$90 per hour, the function would become $y = 90x + 50$. On the other hand, an equation of $y = 8{,}000(1.04)^x$ could model the rising population of a city with 8,000 residents when the annual growth rate is 4%. Students can examine what would happen to the population over 25 years if the rate were 6% instead of 4% or the effect on the equation and function of the city's population were 12,000 instead of 8,000.

Graphs and tables can be used to examine the behaviors of functions as parameters are changed, including the comparison of two functions such as what would happen to a population if it grew by 500 people per year, versus rising an average of 8% per year over the course of 10 years.

Common Misconceptions (from Ohio's Model Curricula):
Students may believe that changing the slope of a linear function from "2" to "3" makes the graph steeper without realizing that there is a real-world context and reason for examining the slopes of lines. Similarly, an exponential function can appear to be abstract until applying it to a real-world situation involving population, cost, investments, etc.

Strategies for Diverse Learners:
Problems of varying difficulty are incorporated into the problem sets.

Literacy Standards Considerations:

- **I can calculate and interpret the average rate of change of a function.**

Word Problems are included as a part of the practice set - students have to interpret into algebraic notation.

- **I can graph a linear function and identify its intercepts.**

Students would need to be familiar with the definitions of key terms.

- **I can demonstrate that a linear function has a constant rate of change.**

Word Problems are included as a part of the practice set - students have to interpret into algebraic notation.

- **I can identify situations in which one quantity changes at a constant rate.**

Word Problems are included as a part of the practice set - students have to interpret into algebraic notation.

- **I can construct linear functions from an arithmetic sequence, graph, table of values, or description of the relationship.**

Word Problems are included as a part of the practice set - students have to interpret into algebraic notation.

- **I can interpret the parameters of a linear function in a real-life problem.**

Word Problems are included as a part of the practice set - students have to interpret into algebraic notation.

Instructional Resources:
Holt McDougal Algebra 1 © 2011

- **I can calculate and interpret the average rate of change of a function.**

https://www.youtube.com/watch?v=avS6C6_kvXM
http://xypi.wordpress.com/2013/07/02/rate-of-change-cards/

- **I can graph a linear function and identify its intercepts.**

http://www.insidemathematics.org/common-core-math-tasks/high-school/HS-A-2006%20Graphs2006.pdf

- **I can demonstrate that a linear function has a constant rate of change.**

http://everybodyisageniusblog.blogspot.com/2013/01/patterns.html
http://exponentialcurve.blogspot.com/2013/09/linear-patterns-in-algebra-1.html

- **I can identify situations in which one quantity changes at a constant rate.**

http://www.insidemathematics.org/common-core-math-tasks/high-school/HS-F-2008%20Functions.pdf

- **I can construct linear functions from an arithmetic sequence, graph, table of values, or description of the relationship.**

http://mathcoachblog.com/2014/03/10/linear-function-stories/
http://graphingstories.com/
http://justtellmetheanswer.wordpress.com/2013/11/06/pizza-functions/
http://blog.mrmeyer.com/2008/linear-fun-2-stacking-cups/
http://everybodyisageniusblog.blogspot.com/2013/02/linear-equations-applications.html
http://www.insidemathematics.org/problems-of-the-month/pom-tritriangles.pdf
http://www.visualpatterns.org
http://www.illustrativemathematics.org/illustrations/243

- **I can interpret the parameters of a linear function in a real-life problem.**

http://untilnextstop.blogspot.com/2010/10/activities-to-help-kids-understand.html

Assessment:

Formative:

- exit slips
- observation of students working on problems in class
- observation of student work from outside of class practice problems

Summative:

- unit test questions

Assessment Resources:

http://www.illustrativemathematics.org/
http://practice.parcc.testnav.com/
IIS
Holt McDougal Algebra 1 © 2011

Unit 4 – Statistical Models

(5 weeks, October, November, December)

Common Core State Standards Addressed:

N-Q.1: Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
N-Q.2: Define appropriate quantities for the purpose of descriptive modeling.
N-Q.3: Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.
S-ID.1: Represent data with plots on the real number line (dot plots, histograms, and box plots).
S-ID.2: Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
S-ID.3: Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).
S-ID.5: Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.
S-ID.6a: Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.
S-ID.6b: Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. Informally assess the fit of a function by plotting and analyzing residuals.
S-ID.6c: Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. Fit a linear function for a scatter plot that suggests a linear association.
S-ID.7: Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
S-ID.8: Compute (using technology) and interpret the correlation coefficient of a linear fit.
S-ID.9: Distinguish between correlation and causation.

Student Friendly Learning Targets (In order of teaching):

- I can describe the center of the data distribution (mean or median).
- I can describe the spread of the data distribution (interquartile range or standard deviation).
- I can represent data with plots on the real number line (dot plots, histograms, and box plots).
- I can compare the distribution of two or more data sets by examining their shapes, centers, and spreads when drawn on the same scale.
- I can interpret the differences in the shape, center, and spread of a data set in the context of a problem, and can account for effects of extreme data points.
- I can read and interpret the data displayed in a two-way frequency table.

- I can interpret and explain the meaning of relative frequencies in the context of a problem.
- I can construct a scatter plot, sketch a line of best fit, and write the equation of that line.
- I can use the function of best fit to make predictions.
- I can analyze the residual plot to determine whether the function is an appropriate fit.
- I can calculate, using technology, and interpret a correlation coefficient.
- I can recognize that correlation does not imply causation and that causation is not illustrated on a scatter plot.

Vocabulary:
dot plot, histogram, box plot, 5-number summary, median, lower quartile, upper quartile, minimum value, maximum value, data, frequency, interval, scale, distribution, shape, center, spread, mean, interquartile range, standard deviation, data distribution, outlier, two-way frequency table, percentages, ratios, relative frequencies, joint relative frequency, marginal relative frequency, conditional relative frequency, bar chart, pie chart, patterns, associations, variables, scatter plot, quantitative variable, independent variable, dependent variable, direction, form, strength, linear, models, function of best fit, line of best fit, residuals, $y = mx + b$, slope, y-intercept, linear equation, linear model, units, correlation coefficient, significance, correlation, causation

Materials and/or Technology Needed:
scientific calculator (TI-30XIIS), graphing calculator, graph paper, ruler, Excel

Instructional Strategies:
Information from all Learning Targets can be found at Ohio's Model Curricula: http://education.ohio.gov/Topics/Academic-Content-Standards/Mathematics and KCTM's Flip Book (which encompasses several states' information including Ohio): http://katm.org/wp/wp-content/uploads/flipbooks/High-School-CCSS-Flip-Book-USD-259-2012.pdf

For all learning targets involving graphing, from the Ohio Model Curricula:
Graphical representations serve as visual models for understanding phenomena that take place in our daily surroundings. The use of different kinds of graphical representations along with their units, labels and titles demonstrate the level of students' understanding and foster the ability to reason, prove, self-check, examine relationships and establish the validity of arguments. Students need to be able to identify misleading graphs by choosing correct units and scales to create a correct representation of a situation or to make a correct conclusion from it.

(From Ohio's Model Curricula) Students need to develop sound mathematical reasoning skills and forms of argument to make reasonable judgments about their solutions. They should be able to decide whether a problem calls for an estimate, for an approximation, or for an exact answer. To accomplish this goal, teachers should provide students with a broad range of contextual problems that offer opportunities for performing operations with quantities involving units. These problems should be connected to science, engineering, economics, finance, medicine, etc.

Students should be able to correctly identify the degree of precision of the answers which should not be far greater than the actual accuracy of the measurements.

For the learning targets on measures of center, measures of spread, and plotting data, from Ohio's Model Curricula:
It is helpful for students to understand that a statistical process is a problem-solving process consisting of four steps: formulating a question that can be answered by data; designing and implementing a plan that collects appropriate data; analyzing the data by graphical and/or numerical methods; and interpreting the analysis in the context of the original question. Opportunities should be provided for students to work through the statistical process. In Grades 6-8, learning has focused on parts of this process. Now is a good time to investigate a problem of interest to the students and follow it through. The richer the question formulated, the more interesting is the process. Teachers and students should make extensive use of resources to perfect this very important first step. Global web resources can inspire projects.

- **I can describe the center of the data distribution (mean or median).**

(From Ohio's Model Curricula) Measures of center and spread for data sets without outliers are the mean and standard deviation, whereas median and interquartile ranges are better measures for data sets with outliers.

- **I can describe the spread of the data distribution (interquartile range or standard deviation).**

(From Ohio's Model Curricula) Measures of center and spread for data sets without outliers are the mean and standard deviation, whereas median and interquartile ranges are better measures for data sets with outliers.

Introduce the formula of standard deviation by reviewing the previously learned MAD (mean absolute deviation). The MAD is very intuitive and gives a solid foundation for developing the more complicated standard deviation measure.

- **I can represent data with plots on the real number line (dot plots, histograms, and box plots).**

(From Ohio's Model Curricula) Have students practice their understanding of the different types of graphs for categorical and numerical variables by constructing statistical posters. Note that a bar graph for categorical data may have frequency on the vertical (student's pizza preferences) or measurement on the vertical (radish root growth over time - days).

Informally observing the extent to which two boxplots or two dotplots overlap begins the discussion of drawing inferential conclusions. Don't shortcut this observation in comparing two data sets.

As histograms for various data sets are drawn, common shapes appear. To characterize the shapes, curves are sketched through the midpoints of the tops of the histogram's rectangles. Of particular importance is a symmetric unimodal curve that has specific areas within one, two, and three standard deviations of its mean. It is called the Normal distribution and students need to be able to find areas (probabilities) for various events using tables or a graphing calculator.

<u>Common Misconceptions (from Ohio's Model Curricula):</u>
<u>Students may believe:</u>
That a bar graph and a histogram are the same. A bar graph is appropriate when the horizontal axis has categories and the vertical axis is labeled by either frequency (e.g., book titles on the horizontal and number of students who like the respective books on the vertical) or measurement of some numerical variable (e.g., days of the week on the horizontal and median length of root growth of radish seeds on the vertical). A histogram has units of measurement of a numerical variable on the horizontal (e.g., ages with intervals of equal length).

That the lengths of the intervals of a boxplot (min,Q1), (Q1,Q2), (Q2,Q3), (Q3,max) are related to the number of subjects in each interval. Students should understand that each interval theoretically contains one-fourth of the total number of subjects. Sketching an accompanying histogram and constructing a live boxplot may help in alleviating this misconception.

- **I can compare the distribution of two or more data sets by examining their shapes, centers, and spreads when drawn on the same scale.**

- **I can interpret the differences in the shape, center, and spread of a data set in the context of a problem, and can account for effects of extreme data points.**

- **I can read and interpret the data displayed in a two-way frequency table.**

(From Ohio's Model Curricula) In the categorical case, begin with two categories for each variable and represent them in a two-way table with the two values of one variable defining the rows and the two values of the other variable defining the columns. (Extending the number of rows and columns is easily done once students become comfortable with the 2x2 case.) The table entries are the joint frequencies of how many subjects displayed the respective cross-classified values. Row totals and column totals constitute the marginal frequencies. Dividing joint or marginal frequencies by the total number of subjects define relative frequencies (and percentages), respectively. Conditional relative frequencies are determined by focusing on a specific row or column of the table. They are particularly useful in determining any associations between the two variables.

- **I can interpret and explain the meaning of relative frequencies in the context of a problem.**

(From Ohio's Model Curricula) In the categorical case, begin with two categories for each variable and represent them in a two-way table with the two values of one variable defining the rows and the two values of the other variable defining the columns. (Extending the number of rows and columns is easily done once students become comfortable with the 2x2 case.) The table entries are the joint frequencies of how many subjects displayed the respective cross-classified values. Row totals and column totals constitute the marginal frequencies. Dividing joint or marginal frequencies by the total number of subjects define relative frequencies (and percentages), respectively. Conditional relative frequencies are determined by focusing on a specific row or column of the table. They are particularly useful in determining any associations between the two variables.

- **I can construct a scatter plot, sketch a line of best fit, and write the equation of that line.**

(From Ohio's Model Curricula) In the numerical or quantitative case, display the paired data in a scatterplot. Note that although the two variables in general will not have the same scale, e.g., total SAT versus grade-point average, it is best to begin with variables with the same scale such as SAT Verbal and SAT Math. Fitting functions to such data will avoid difficulties such as interpretation of slope in the linear case in which scales differ. Once students are comfortable with the same scale case, introducing different scales situations will be less problematic.

<u>Common Misconceptions from Ohio's Model Curricula:</u>
<u>Students may believe:</u>
That a 45 degree line in the scatterplot of two numerical variables always indicates a slope of 1 which is the case only when the two variables have the same scaling.

- **I can use the function of best fit to make predictions.**

(From Ohio's Model Curricula) In the numerical or quantitative case, display the paired data in a scatterplot. Note that although the two variables in general will not have the same scale, e.g., total SAT versus grade-point average, it is best to begin with variables with the same scale such as SAT Verbal and SAT Math. Fitting functions to such data will avoid difficulties such as interpretation of slope in the linear case in which scales differ. Once students are comfortable with the same scale case, introducing different scales situations will be less problematic.

<u>Common Misconceptions from Ohio's Model Curricula:</u>
<u>Students may believe:</u>
That a 45 degree line in the scatterplot of two numerical variables always indicates a slope of 1 which is the case only when the two variables have the same scaling.
Those residual plots in the quantitative case should show a pattern of some sort. Just the opposite is the case.

- **I can analyze the residual plot to determine whether the function is an appropriate fit.**

(From Ohio's Model Curricula) In the numerical or quantitative case, display the paired data in a scatterplot. Note that although the two variables in general will not have the same scale, e.g., total SAT versus grade-point average, it is best to begin with variables with the same scale such as SAT Verbal and SAT Math. Fitting functions to such data will avoid difficulties such as interpretation of slope in the linear case in which scales differ. Once students are comfortable with the same scale case, introducing different scales situations will be less problematic.

<u>Common Misconceptions from Ohio's Model Curricula:</u>
<u>Students may believe:</u>
Those residual plots in the quantitative case should show a pattern of some sort. Just the opposite is the case.

- **I can calculate, using technology, and interpret a correlation coefficient.**

(From Ohio's Model Curricula) To make some sense of Pearson's r, correlation coefficient, students should recall their middle school experience with the Quadrant Count Ratio (QCR) as a measure of relationship between two quantitative variables.

- **I can recognize that correlation does not imply causation and that causation is not illustrated on a scatter plot.**

(From Ohio's Model Curricula) Noting that a correlated relationship between two quantitative variables is not causal (unless the variables are in an experiment) is a very important topic and a substantial amount of time should be spent on it.

<u>Common Misconceptions from Ohio's Model Curricula:</u>
That when two quantitative variables are related, i.e., correlated, that one causes the other to occur. Causation is not necessarily the case. For example, at a theme park, the daily temperature and number of bottles of water sold are demonstrably correlated, but an increase in the number of bottles of water sold does not cause the day's temperature to rise or fall.

<u>**Strategies for Diverse Learners:**</u>
Problems of varying difficulty are incorporated into the problem sets.

<u>**Literacy Standards Considerations:**</u>

- **I can describe the center of the data distribution (mean or median).**

Students would need to be familiar with the definitions of key terms.

- **I can describe the spread of the data distribution (interquartile range or standard deviation).**

Students would need to be familiar with the definitions of key terms.

- **I can represent data with plots on the real number line (dot plots, histograms, and box plots).**

Students would need to be familiar with the definitions of key terms.

- **I can compare the distribution of two or more data sets by examining their shapes, centers, and spreads when drawn on the same scale.**

Students would need to be familiar with the definitions of key terms. Word Problems are included as a part of the practice set - students will have to interpret the key terms as they apply to the problem.

- **I can interpret the differences in the shape, center, and spread of a data set in the context of a problem, and can account for effects of extreme data points.**

Students would need to be familiar with the definitions of key terms. Word Problems are included as a part of the practice set - students will have to interpret the key terms as they apply to the problem.

- **I can interpret the differences in the shape, center, and spread of a data set in the context of a problem, and can account for effects of extreme data points.**

Students would need to be familiar with the definitions of key terms. Word Problems are included as a part of the practice set - students will have to interpret the key terms as they apply to the problem.

- **I can read and interpret the data displayed in a two-way frequency table.**

Students would need to be familiar with the definitions of key terms. Word Problems are included as a part of the practice set - students will have to interpret the key terms as they apply to the problem.

- **I can interpret and explain the meaning of relative frequencies in the context of a problem.**

Students would need to be familiar with the definitions of key terms. Word Problems are included as a part of the practice set - students will have to interpret the key terms as they apply to the problem.

- **I can construct a scatter plot, sketch a line of best fit, and write the equation of that line.**

Students would need to be familiar with the definitions of key terms.

- **I can use the function of best fit to make predictions.**

Students would need to be familiar with the definitions of key terms. Word Problems are included as a part of the practice set - students will have to interpret the key terms as they apply to the problem.

- **I can analyze the residual plot to determine whether the function is an appropriate fit.**

Students would need to be familiar with the definitions of key terms.

- **I can calculate, using technology, and interpret a correlation coefficient.**

Students would need to be familiar with the definitions of key terms.

- **I can recognize that correlation does not imply causation and that causation is not illustrated on a scatter plot.**

Students would need to be familiar with the definitions of key terms. Word Problems are included as a part of the practice set - students will have to interpret the key terms as they apply to the problem.

Instructional Resources:

Holt McDougal Algebra 1 © 2011

- **I can describe the center of the data distribution (mean or median).**
- **I can describe the spread of the data distribution (interquartile range or standard deviation).**
- **I can represent data with plots on the real number line (dot plots, histograms, and box plots).**
- **I can compare the distribution of two or more data sets by examining their shapes, centers, and spreads when drawn on the same scale.**
- **I can interpret the differences in the shape, center, and spread of a data set in the context of a problem, and can account for effects of extreme data points.**
- **I can read and interpret the data displayed in a two-way frequency table.**
- **I can interpret and explain the meaning of relative frequencies in the context of a problem.**
- **I can construct a scatter plot, sketch a line of best fit, and write the equation of that line.**

http://mathequalslove.blogspot.com/2013/10/fun-with-linear-regression-labs.html

- **I can use the function of best fit to make predictions.**
- **I can analyze the residual plot to determine whether the function is an appropriate fit.**
- **I can calculate, using technology, and interpret a correlation coefficient.**
- **I can recognize that correlation does not imply causation and that causation is not illustrated on a scatter plot.**

Assessment:

Formative:

- exit slips

- observation of students working on problems in class
- observation of student work from outside of class practice problems

Summative:
- unit test questions

Assessment Resources:
http://www.illustrativemathematics.org/
http://practice.parcc.testnav.com/
IIS
Holt McDougal Algebra 1 © 2011

Unit 5 – Linear Equations and Inequalities

(4 weeks, December, January)

Common Core State Standards Addressed (In order of teaching):

A-CED.1: Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions*.

A-CED.3: Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods*.

A-CED.4: Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law $V = IR$ to highlight resistance R*.

A-REI.1: Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

A-REI.3: Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

A-REI.12: Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Student Friendly Learning Targets:

- I can solve a linear equation in one variable.
- I can solve a linear inequality in one variable.
- I can solve formulas for a specified variable.
- I can solve an absolute value equation in one variable.
- I can solve and graph a compound inequality in one variable.
- I can solve an absolute value inequality in one variable.
- I can graph a linear inequality on a coordinate plane.

Vocabulary:

linear, linear equation, linear inequality, coefficient, constraints, solutions, formula, literal equation, absolute value, compound inequality, and, or, intersection, union, half-plane, boundary, coordinate plane

Materials and/or Technology Needed:

calculator, graphing calculator, Desmos.com, graph paper, ruler

Instructional Strategies:

Information from all Learning Targets can be found at Ohio's Model Curricula: http://education.ohio.gov/Topics/Academic-Content-Standards/Mathematics and KCTM's Flip

Book (which encompasses several states' information including Ohio): http://katm.org/wp/wp-content/uploads/flipbooks/High-School-CCSS-Flip-Book-USD-259-2012.pdf

- **I can solve a linear equation in one variable.**

(From Ohio's Model Curricula) Provide examples of real-world problems that can be modeled by writing an equation or inequality. Begin with simple equations and inequalities and build up to more complex equations in two or more variables that may involve quadratic, exponential or rational functions.

Provide examples of real-world problems that can be solved by writing an equation, and have students explore the graphs of the equations on a graphing calculator to determine which parts of the graph are relevant to the problem context.

Challenge students to justify each step of solving an equation. Transforming $2x - 5 = 7$ to $2x = 12$ is possible because 5 = 5, so adding the same quantity to both sides of an equation makes the resulting equation true as well. Each step of solving an equation can be defended, much like providing evidence for steps of a geometric proof.

Provide examples for how the same equation might be solved in a variety of ways as long as equivalent quantities are added or subtracted to both sides of the equation, the order of steps taken will not matter.

$3n + 2 = n - 10$		$3n + 2 = n - 10$		$3n + 2 = n - 10$
$-\quad 2 = \quad -2$		$+\quad 10 = \quad +10$		$-n \quad = -n$
$3n \quad = n - 12$	OR	$3n + 12 = n$	OR	$2n + 2 = -10$
$-n \quad = -n$		$-3n \quad = -3n$		$-2 = -2$
$2n \quad = -12$		$12 = -2n$		$2n = -12$
$n = -6$		$n = -6$		$n = -6$

Begin with simple, one-step equations and require students to write out a justification for each step used to solve the equation.

There are two major reasons for discussing the topic of inequalities along with equations: First, there are analogies between solving equations and inequalities that help students understand them both**.** Second, the applications that lead to equations almost always lead in the same way to inequalities.

In grades 6-8, students solve and graph linear equations and inequalities. It may be beneficial to remind students of the most common solving techniques, such as converting fractions from one form to another, removing parentheses in the sentences, or multiplying both sides of an equation or inequality by the common denominator of the fractions.

Careful selection of examples and exercises is needed to provide students with meaningful review and to introduce other important concepts, such as the use of properties and applications of solving linear equations and inequalities. Stress the idea that the application of properties is also appropriate when working with equations or inequalities that include more than one variable, fractions and decimals. Regardless of the type of numbers or variables in the equation or inequality, students have to examine the validity of each step in the solution process.

<u>Common Misconceptions from Ohio's Model Curricula:</u>
Students may believe that equations of linear, quadratic and other functions are abstract and exist only "in a math book," without seeing the usefulness of these functions as modeling real-world phenomena.

Students may believe that solving an equation such as $3x+1=7$ involves "only removing the 1," failing to realize that the equation 1 = 1 is being subtracted to produce the next step.

Some students may believe that for equations containing fractions only on one side, it requires "clearing fractions" (the use of multiplication) only on that side of the equation. To address this misconception, start by demonstrating the solution methods for equations similar to $\frac{1}{4}x + \frac{1}{5}x + \frac{1}{6}x + 46 = x$ and stress that the Multiplication Property of Equality is applied to both sides, which are multiplied by 60.

- **I can solve a linear inequality in one variable.**

(From Ohio's Model Curricula) Provide examples of real-world problems that can be modeled by writing an equation or inequality. Begin with simple equations and inequalities and build up to more complex equations in two or more variables that may involve quadratic, exponential or rational functions.

There are two major reasons for discussing the topic of inequalities along with equations: First, there are analogies between solving equations and inequalities that help students understand them both. Second, the applications that lead to equations almost always lead in the same way to inequalities.

In grades 6-8, students solve and graph linear equations and inequalities. Graphing experience with inequalities is limited to graphing on a number line diagram. Despite this work, some students will still need more practice to be proficient. It may be beneficial to remind students of the most common solving techniques, such as converting fractions from one form to another, removing parentheses in the sentences, or multiplying both sides of an equation or inequality by the common denominator of the fractions. Students must be aware of what it means to check an inequality's solution. The substitution of the end points of the solution set in the original inequality should give equality regardless of the presence or the absence of an equal sign in the original sentence. The substitution of any value from the rest of the solution set should give a correct inequality.

Careful selection of examples and exercises is needed to provide students with meaningful review and to introduce other important concepts, such as the use of properties and applications of solving linear equations and inequalities. Stress the idea that the application of properties is also appropriate when working with equations or inequalities that include more than one variable, fractions and decimals. Regardless of the type of numbers or variables in the equation or inequality, students have to examine the validity of each step in the solution process.

Common Misconceptions from Ohio's Model Curricula:
Students may confuse the rule of changing a sign of an inequality when multiplying or dividing by a negative number with changing the sign of an inequality when one or two sides of the inequality become negative (for ex., $3x > -15$ or $x < -5$).

- **I can solve formulas for a specified variable.**

(From Ohio's Model Curricula) Explore examples illustrating when it is useful to rewrite a formula by solving for one of the variables in the formula. For example, the formula for the area of a trapezoid ($A = \frac{1}{2}h(b_1 + b_2)$) can be solved for *h* if the area and lengths of the bases are known but the height needs to be calculated. This strategy of selecting a different representation has many applications in science and business when using formulas.

Give students formulas, such as area and volume (or from science or business), and have students solve the equations for each of the different variables in the formula.

There are two major reasons for discussing the topic of inequalities along with equations: First, there are analogies between solving equations and inequalities that help students understand them both. Second, the applications that lead to equations almost always lead in the same way to inequalities.

Solving equations for the specified letter with coefficients represented by letters (e.g.,$A = \frac{1}{2}h(B + b)$, when solving for b) is similar to solving an equation with one variable. Provide students with an opportunity to abstract from particular numbers and apply the same kind of manipulations to formulas as they did to equations. One of the purposes of doing abstraction is to learn how to evaluate the formulas in easier ways and use the techniques across mathematics and science.

Draw students' attention to equations containing variables with subscripts. The same variables with different subscripts (e.g., x_1 and x_2) should be viewed as different variables that cannot be combined as like terms. A variable with a variable subscript, such as a_n, must be treated as a single variable – the *n*th term, where variables *a* and *n* have different meaning.

Common Misconceptions from Ohio's Model Curricula:
Students may believe that equations of linear, quadratic and other functions are abstract and exist only "in a math book," without seeing the usefulness of these functions as modeling real-world phenomena.

Some students may believe that subscripts can be combined as $b_1 + b_2 = b_3$ and the sum of different variables *d* and D is 2D (*d* +D = 2D).

- **I can solve an absolute value equation in one variable.**

(From Ohio's Model Curricula) Challenge students to justify each step of solving an equation. Transforming $2x - 5 = 7$ to $2x = 12$ is possible because 5 = 5, so adding the same quantity to both sides of an equation makes the resulting equation true as well. Each step of solving an equation can be defended, much like providing evidence for steps of a geometric proof.

Provide examples for how the same equation might be solved in a variety of ways as long as equivalent quantities are added or subtracted to both sides of the equation, the order of steps taken will not matter.

$3n + 2 = n - 10$		$3n + 2 = n - 10$		$3n + 2 = n - 10$
$-\quad 2 = \quad -2$		$+\quad 10 = \quad +10$		$-n \quad = -n$
$3n \quad = n - 12$	OR	$3n + 12 = n$	OR	$2n + 2 = -10$
$-n \quad = -n$		$-3n \quad = -3n$		$-2 = -2$
$2n \quad = -12$		$12 = -2n$		$2n = -12$
$n = -6$		$n = -6$		$n = -6$

Begin with simple, one-step equations and require students to write out a justification for each step used to solve the equation.

There are two major reasons for discussing the topic of inequalities along with equations: First, there are analogies between solving equations and inequalities that help students understand them both**.** Second, the applications that lead to equations almost always lead in the same way to inequalities.

In grades 6-8, students solve and graph linear equations and inequalities. Graphing experience with inequalities is limited to graphing on a number line diagram. Despite this work, some students will still need more practice to be proficient. It may be beneficial to remind students of the most common solving techniques, such as converting fractions from one form to another, removing parentheses in the sentences, or multiplying both sides of an equation or inequality by the common denominator of the fractions.

Careful selection of examples and exercises is needed to provide students with meaningful review and to introduce other important concepts, such as the use of properties and applications of solving linear equations and inequalities. Stress the idea that the application of properties is also appropriate when working with equations or inequalities that include more than one variable, fractions and decimals. Regardless of the type of numbers or variables in the equation or inequality, students have to examine the validity of each step in the solution process.

<u>Common Misconceptions from Ohio's Model Curricula:</u>

Students may believe that solving an equation such as $3x+1=7$ involves "only removing the 1," failing to realize that the equation 1 = 1 is being subtracted to produce the next step.

- **I can solve and graph a compound inequality in one variable.**

(From Ohio's Model Curricula) There are two major reasons for discussing the topic of inequalities along with equations: First, there are analogies between solving equations and inequalities that help students understand them both**.** Second, the applications that lead to equations almost always lead in the same way to inequalities.

In grades 6-8, students solve and graph linear equations and inequalities. Graphing experience with inequalities is limited to graphing on a number line diagram. Despite this work, some students will still need more practice to be proficient. It may be beneficial to remind students of the most common solving techniques, such as converting fractions from one form to another, removing parentheses in the sentences, or multiplying both sides of an equation or inequality by the common denominator of the fractions. Students must be aware of what it means to check an inequality's solution. The substitution of the end points of the solution set in the original inequality should give equality regardless of the presence or the absence of an equal sign in the original sentence**.** The substitution of any value from the rest of the solution set should give a correct inequality.

Careful selection of examples and exercises is needed to provide students with meaningful review and to introduce other important concepts, such as the use of properties and applications of solving linear equations and inequalities. Stress the idea that the application of properties is also appropriate when working with equations or inequalities that include more than one variable, fractions and decimals. Regardless of the type of numbers or variables in the equation or inequality, students have to examine the validity of each step in the solution process.

<u>Common Misconceptions from Ohio's Model Curricula:</u>
Students may confuse the rule of changing a sign of an inequality when multiplying or dividing by a negative number with changing the sign of an inequality when one or two sides of the inequality become negative (for ex., $3x > -15$ or $x < -5$).

- **I can solve an absolute value inequality in one variable.**

(From Ohio's Model Curricula) There are two major reasons for discussing the topic of inequalities along with equations: First, there are analogies between solving equations and inequalities that help students understand them both**.** Second, the applications that lead to equations almost always lead in the same way to inequalities.

In grades 6-8, students solve and graph linear equations and inequalities. Graphing experience with inequalities is limited to graphing on a number line diagram. Despite this work, some students will still need more practice to be proficient. It may be beneficial to remind students of the most common solving techniques, such as converting fractions from one form to another, removing parentheses in the sentences, or multiplying both sides of an equation or inequality by the common denominator of the fractions. Students must be aware of what it means to check an inequality's solution. The substitution of the end points of the solution set in the original inequality should give equality regardless of the presence or the absence of an equal sign in the original sentence**.** The substitution of any value from the rest of the solution set should give a correct inequality.

Careful selection of examples and exercises is needed to provide students with meaningful review and to introduce other important concepts, such as the use of properties and applications of solving linear equations and inequalities. Stress the idea that the application of properties is also appropriate when working with equations or inequalities that include more than one variable, fractions and decimals. Regardless of the type of numbers or variables in the equation or inequality, students have to examine the validity of each step in the solution process.

Common Misconceptions from Ohio's Model Curricula:
Students may confuse the rule of changing a sign of an inequality when multiplying or dividing by a negative number with changing the sign of an inequality when one or two sides of the inequality become negative (for ex., $3x > -15$ or $x < -5$).

- **I can graph a linear inequality on a coordinate plane.**

(From Ohio's Model Curricula) Investigate real-world examples of two-dimensional inequalities. For example, students might explore what the graph would look like for money earned when a person earns *at least* $6 per hour. (The graph for a person earning *exactly* $6/hour would be a linear function, while the graph for a person earning at least $6/hour would be a half-plane including the line and all points above it.)

Common Misconceptions from Ohio's Model Curricula:
Students may believe that the graph of a function is simply a line or curve "connecting the dots," without recognizing that the graph represents all solutions to the equation.

Students may also believe that graphing linear and other functions is an isolated skill, not realizing that multiple graphs can be drawn to solve equations involving those functions.

Additionally, students may believe that two-variable inequalities have no application in the real world. Teachers can consider business related problems (e.g., linear programming applications) to engage students in discussions of how the inequalities are derived and how the feasible set includes all the points that satisfy the conditions stated in the inequalities.

Strategies for Diverse Learners:
Problems of varying difficulty are incorporated into the problem sets.

Literacy Standards Considerations:

- **I can solve a linear equation in one variable.**

Word Problems are included as a part of the practice set - students have to interpret into algebraic notation.

- **I can solve a linear inequality in one variable.**

Word Problems are included as a part of the practice set - students have to interpret into algebraic notation.

- **I can solve formulas for a specified variable.**

Word Problems are included as a part of the practice set - students have to interpret into algebraic notation.

- **I can solve an absolute value equation in one variable.**

Word Problems are included as a part of the practice set - students have to interpret into algebraic notation.

- **I can solve and graph a compound inequality in one variable.**

Word Problems are included as a part of the practice set - students have to interpret into algebraic notation.

- **I can solve an absolute value inequality in one variable.**

Word Problems are included as a part of the practice set - students have to interpret into algebraic notation.

- **I can graph a linear inequality on a coordinate plane.**

Word Problems are included as a part of the practice set - students have to interpret into algebraic notation.

Instructional Resources:
Holt McDougal Algebra 1 © 2011

- **I can solve a linear equation in one variable.**

http://exponentialcurve.blogspot.com/2013/10/constructing-and-deconstructing.html
http://map.mathshell.org/materials/lessons.php?taskid=487&subpage=concept
http://justtellmetheanswer.wordpress.com/2013/08/30/using-common-core-you-need-desmos/ (first topic)
http://everybodyisageniusblog.blogspot.com/2012/07/solving-special-case-equations.html

- **I can solve a linear inequality in one variable.**

- **I can solve formulas for a specified variable.**

- **I can solve an absolute value equation in one variable.**

http://function-of-time.blogspot.com/2011/09/algebra-2-solving-absolute-value.html

- **I can solve and graph a compound inequality in one variable.**

- **I can solve an absolute value inequality in one variable.**

- **I can graph a linear inequality on a coordinate plane.**

http://mrshester.blogspot.com/2014/03/making-sense-of-linear-inequalities.html

Assessment:

Formative:

- exit slips
- observation of students working on problems in class
- observation of student work from outside of class practice problems

Summative:

- unit test questions

Assessment Resources:

http://www.illustrativemathematics.org/
http://practice.parcc.testnav.com/
IIS
Holt McDougal Algebra 1 © 2011

Unit 6 – Systems of Linear Equations and Inequalities

(2 ½ weeks, January, February)

Common Core State Standards Addressed:

A-CED.3: Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods*.

A-REI.5: Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

A-REI.6: Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

A-REI.11: Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.*

A-REI.12: Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Student Friendly Learning Targets (In order of teaching):

- I can solve a system of linear equations by graphing.
- I can solve a system of linear equations by substitution.
- I can solve a system of linear equations by the elimination method.
- I can solve a system of linear inequalities by graphing.
- I can write and graph a set of constraints for a linear-programming problem and find the maximum and/or minimum values.

Vocabulary:

linear equation, linear inequality, coefficient, system of equations, equivalent equations, elimination method, substitution method, solution of a system, intersection, x-coordinate, y-coordinate, linear function, system of linear inequalities, half-plane, boundary, coordinate plane, linear inequality

Materials and/or Technology Needed:

calculator, graphing calculator, Desmos.com, graph paper, ruler

Instructional Strategies:

Information from all Learning Targets can be found at Ohio's Model Curricula: http://education.ohio.gov/Topics/Academic-Content-Standards/Mathematics and KCTM's Flip

Book (which encompasses several states' information including Ohio): http://katm.org/wp/wp-content/uploads/flipbooks/High-School-CCSS-Flip-Book-USD-259-2012.pdf

- **I can solve a system of linear equations by graphing.**
- **I can solve a system of linear equations by substitution.**
- **I can solve a system of linear equations by the elimination method.**

(For all three from Ohio's Model Curricula) Connect the idea of adding two equations together as a means of justifying steps of solving a simple equation to the process of solving a system of equations. A system consisting of two linear functions such as $2x + 3y = 8$ and $x - 3y = 1$ can be solved by adding the equations together, and can be justified by exactly the same reason that solving the equation $2x - 4 = 5$ can begin by adding the equation 4 = 4.

The focus of this standard is to provide mathematics justification for the addition (elimination) and substitution methods of solving systems of equations that transform a given system of two equations into a simpler equivalent system that has the same solutions as the original.

The Addition and Multiplication Properties of Equality allow finding solutions to certain systems of equations. In general, any linear combination, $m(Ax + By) + n(Cx + Dy) = mE + nF$, of two linear equations

$Ax + By = E$ and
$Cx + Dy = F$

intersecting in a single point contains that point. The multipliers m and n can be chosen so that the resulting combination has only an x-term or only a y-term in it. That is, the combination will be a horizontal or vertical line containing the point of intersection.

In the proof of a system of two equations in two variables, where one equation is replaced by the sum of that equation and a multiple of the other equation, produces a system that has the same solutions, let point (x_1, y_1) be a solution of both equations:

$Ax_1 + By_1 = E$ (true)
$Cx_1 + Dy_1 = F$ (true)

Replace the equation $Ax + By = E$ with $Ax + By + k(Cx + Dy)$ on its left side and with $E + kF$ on its right side.

The new equation is $Ax + By + k(Cx + Dy) = E + kF$.

Show that the ordered pair of numbers (x_1, y_1) is a solution of this equation by replacing (x_1, y_1) in the left side of this equation and verifying that the right side really equals $E + kF$:

$$Ax_1 + By_1 + k(Cx_1 + Dy_1) = E + kF \text{ (true)}$$

Systems of equations are classified into two groups, consistent or inconsistent, depending on whether or not solutions exist. The solution set of a system of equations is the intersection of the solution sets for the individual equations. Stress the benefit of making the appropriate selection of a method for solving systems (graphing vs. addition vs. substitution). This depends on the type of equations and combination of coefficients for corresponding variables, without giving a preference to either method.

The graphing method can be the first step in solving systems of equations. .A set of points representing solutions of each equation is found by graphing these equations. Even though the graphing method is limited in finding exact solutions and often yields approximate values, the use of it helps to discover whether solutions exist and, if so, how many are there

Prior to solving systems of equations graphically, students should revisit "families of functions" to review techniques for graphing different classes of functions. Alert students to the fact that if one equation in the system can be obtained by multiplying both sides of another equation by a nonzero constant, the system is called consistent, the equations in the system are called dependent and the system has an infinite number of solutions that produces coinciding graphs. Provide students opportunities to practice linear vs. non-linear systems; consistent vs. inconsistent systems; pure computational vs. real-world contextual problems (e.g., chemistry and physics applications encountered in science classes). A rich variety of examples can lead to discussions of the relationships between coefficients and consistency that can be extended to graphing and later to determinants and matrices.

The next step is to turn to algebraic methods, elimination or substitution, to allow students to find exact solutions. For any method, stress the importance of having a well-organized format for writing solutions.

Explore visual ways to solve an equation such as $2x + 3 = x – 7$ by graphing the functions $y = 2x + 3$ and $y = x – 7$. Students should recognize that the intersection point of the lines is at (-10, -17). They should be able to verbalize that the intersection point means that when $x = -10$ is substituted into both sides of the equation, each side simplifies to a value of -17. Therefore, -10 is the solution to the equation. This same approach can be used whether the functions in the original equation are linear, nonlinear or both.

Using technology, have students graph a function and use the trace function to move the cursor along the curve. Discuss the meaning of the ordered pairs that appear at the bottom of the calculator, emphasizing that every point on the curve represents a solution to the equation. Begin with simple linear equations and how to solve them using the graphs and tables on a graphing calculator.

Use the table function on a graphing calculator to solve equations. For example, to solve the equation $x^2 = x + 12$, students can examine the equations $y = x^2$ *and* $y = x + 12$ and determine that they intersect when $x = 4$ and when $x = -3$ by examining the table to find where the *y*-values are the same.

Common Misconceptions from Ohio's Model Curricula:

Students may also believe that graphing linear and other functions is an isolated skill, not realizing that multiple graphs can be drawn to solve equations involving those functions.

- **I can solve a system of linear inequalities by graphing.**
- **I can write and graph a set of constraints for a linear-programming problem and find the maximum and/or minimum values.**

(For both from Ohio's Model Curricula) Investigate real-world examples of two-dimensional inequalities. For example, students might explore what the graph would look like for money earned when a person earns *at least* \$6 per hour. (The graph for a person earning *exactly* \$6/hour would be a linear function, while the graph for a person earning at least \$6/hour would be a half-plane including the line and all points above it.) Applications such as linear programming can help students to recognize how businesses use constraints to maximize profit while minimizing the use of resources. These situations often involve the use of systems of two variable inequalities.

<u>Common Misconceptions from Ohio's Model Curricula:</u>
Students may also believe that graphing linear and other functions is an isolated skill, not realizing that multiple graphs can be drawn to solve equations involving those functions.

Additionally, students may believe that two-variable inequalities have no application in the real world. Teachers can consider business related problems (e.g., linear programming applications) to engage students in discussions of how the inequalities are derived and how the feasible set includes all the points that satisfy the conditions stated in the inequalities.

<u>Strategies for Diverse Learners:</u>
Problems of varying difficulty are incorporated into the problem sets.

<u>Literacy Standards Considerations:</u>
- **I can solve a system of linear equations by graphing.**

Word Problems are included as a part of the practice set - students have to interpret into algebraic notation.

- **I can solve a system of linear equations by substitution.**

Word Problems are included as a part of the practice set - students have to interpret into algebraic notation.

- **I can solve a system of linear equations by the elimination method.**

Word Problems are included as a part of the practice set - students have to interpret into algebraic notation.

- **I can solve a system of linear inequalities by graphing.**

Word Problems are included as a part of the practice set - students have to interpret into algebraic notation.

- **I can write and graph a set of constraints for a linear-programming problem and find the maximum and/or minimum values.**

Word Problems are included as a part of the practice set - students have to interpret into algebraic notation.

Instructional Resources:

Holt McDougal Algebra 1 © 2011

- **I can solve a system of linear equations by graphing.**

- **I can solve a system of linear equations by substitution.**

http://cheesemonkeysf.blogspot.com/2013/05/substitution-with-stars.html

- **I can solve a system of linear equations by the elimination method.**

- **I can solve a system of linear inequalities by graphing.**

http://infinitesums.com/commentary/2012/9/23/linear-inequality-exploration.html
http://infinitesums.com/commentary/2013/11/8/more-inequality-explorations
http://mrshester.blogspot.com/2014/03/making-sense-of-linear-inequalities.html

- **I can write and graph a set of constraints for a linear-programming problem and find the maximum and/or minimum values.**

http://www.illustrativemathematics.org/illustrations/610
http://www.illustrativemathematics.org/illustrations/1351
http://fawnnguyen.com/2012/12/31/lego-pieces-and-feasible-region.aspx

Assessment:

Formative:

- exit slips
- observation of students working on problems in class
- observation of student work from outside of class practice problems

Summative:

- unit test questions

Assessment Resources:

http://www.illustrativemathematics.org/
http://practice.parcc.testnav.com/
IIS
Holt McDougal Algebra 1 © 2011

Unit 7 – Polynomial Expressions and Functions

(4 weeks, February, March)

Common Core State Standards Addressed:

A-SSE.1b: Interpret expressions that represent a quantity in terms of its context.* Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.*

A-SSE.2: Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.*

A-SSE.3a: Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.* Factor a quadratic expression to reveal the zeroes of the function it defines.

A-APR.1: Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

A-APR.3: Identify zeroes of polynomials when suitable factorizations are available, and use the zeroes to construct a rough graph of the function defined by the polynomial.

Student Friendly Learning Targets (In order of teaching):

- I can add and subtract polynomials.
- I can multiply polynomials.
- I can rewrite an expression using factoring.
- I can solve quadratic equations by factoring.
- I can sketch a rough graph using the zeroes of a quadratic function and other easily identifiable points.

Vocabulary:

expression, term, factor, coefficient, equivalent, zeroes, x-intercept, y-intercept, domain, range

Materials and/or Technology Needed:

calculator, graph paper, graphing calculator, Desmos.com

Instructional Strategies:

Information from all Learning Targets can be found at Ohio's Model Curricula: http://education.ohio.gov/Topics/Academic-Content-Standards/Mathematics and KCTM's Flip Book (which encompasses several states' information including Ohio): http://katm.org/wp/wp-content/uploads/flipbooks/High-School-CCSS-Flip-Book-USD-259-2012.pdf

- **I can add and subtract polynomials.**
- **I can multiply polynomials.**

(For both from Ohio's Model Curricula) The primary strategy for this cluster is to make connections between arithmetic of integers and arithmetic of polynomials. In order to understand this standard, students need to work toward both understanding and fluency with polynomial arithmetic. Furthermore, to talk about their work, students will need to use correct vocabulary, such as integer, monomial, polynomial, factor, and term.

In arithmetic of polynomials, a central idea is the distributive property, because it is fundamental not only in polynomial multiplication but also in polynomial addition and subtraction. With the distributive property, there is little need to emphasize misleading mnemonics, such as FOIL, which is relevant only when multiplying two binomials, and the procedural reminder to "collect like terms" as a consequence of the distributive property. For example, when adding the polynomials 3x and 2x, the result can be explained with the distributive property as follows: 3x + 2x = (3 + 2)x = 5x.

An important connection between the arithmetic of integers and the arithmetic of polynomials can be seen by considering whole numbers in base ten place value to be polynomials in the base *b* = 10. For two-digit whole numbers and linear binomials, this connection can be illustrated with area models and algebra tiles. But the connections between methods of multiplication can be generalized further. For example, compare the product 213 x 47 with the product $(2b^2 + 1b + 3)(4b + 7)$:

$$\begin{array}{rrrr} & 2b^2 & + \ 1b & + \ 3 \\ \times & & 4b & + \ 7 \\ \hline & 14b^2 & + \ 7b & + \ 21 \\ 8b^3 \ + & 4b^2 & + \ 12b & \\ \hline 8b^3 \ + & 18b^2 & + \ 19b & + \ 21 \end{array} \qquad \begin{array}{rrrr} & 200 \ + & 10 \ + & 3 \\ \times & & 40 \ + & 7 \\ \hline & 1400 \ + & 70 \ + & 21 \\ 8000 \ + & 400 \ + & 120 & \\ \hline 8000 \ + & 1800 \ + & 190 \ + & 21 \end{array} \qquad \begin{array}{rr} & 213 \\ \times & 47 \\ \hline & 1491 \\ & 8520 \\ \hline & 10011 \end{array}$$

Note how the distributive property is in play in each of these examples: In the left-most computation, each term in the factor $(4b + 7)$ must be multiplied by each term in the other factor, $(2b^2 + 1b + 3)$, just like the value of each digit in 47 must be multiplied by the value of each digit in 213, as in the middle computation, which is similar to "partial products methods" that some students may have used for multiplication in the elementary grades. The common algorithm on the right is merely an abbreviation of the partial products method.

The new idea in this standard is called *closure*: A set is *closed* under an operation if when any two elements are combined with that operation, the result is always another element of the same set. In order to understand that polynomials are closed under addition, subtraction and multiplication, students can compare these ideas with the analogous claims for integers: The sum, difference or product of any two integers is an integer, but the quotient of two integers is not always an integer.

Now for polynomials, students need to reason that the sum (difference or product) of any two polynomials is indeed a polynomial. At first, restrict attention to polynomials with integer coefficients. Later, students should consider polynomials with rational or real coefficients and reason that such polynomials are closed under these operations.

For contrast, students need to reason that polynomials are not closed under the operation of division: The quotient of two polynomials is not always a polynomial. For example $(x^2 + 1) \div x$ is not a polynomial. Of course, the quotient of two polynomials is sometimes a polynomial. For example,$(x^2 - 9) \div (x - 3) = x + 3$.

- **I can rewrite an expression using factoring.**

(From Ohio's Model Curricula) Factoring by grouping is another example of how students might analyze the structure of an expression. To factor $3x(x – 5) + 2(x – 5)$, students should recognize that the "$x – 5$" is common to both expressions being added, so it simplifies to $(3x + 2)(x – 5)$. Students should become comfortable with rewriting expressions in a variety of ways until a structure emerges.

Have students create their own expressions that meet specific criteria (e.g., number of terms factorable, difference of two squares, etc.) and verbalize how they can be written and rewritten in different forms. Additionally, pair/group students to share their expressions and rewrite one another's expressions.

Technology may be useful to help a student recognize that two different expressions represent the same relationship. For example, since $(x – y)(x + y)$ can be rewritten as $x^2 – y^2$, they can put both expressions into a graphing calculator (or spreadsheet) and have it generate two tables (or two columns of one table), displaying the same output values for each expression.

This cluster focuses on linking expressions and functions, i.e., creating connections between multiple representations of functional relations – the dependence between a quadratic expression and a graph of the quadratic function it defines, and the dependence between different symbolic representations of exponential functions. Teachers need to foster the idea that changing the forms of expressions, such as factoring or completing the square, or transforming expressions from one exponential form to another, are not independent algorithms that are learned for the sake of symbol manipulations. They are processes that are guided by goals (e.g., investigating properties of families of functions and solving contextual problems).

<u>Common Misconceptions from Ohio's Model Curricula:</u>
Students may believe that the use of algebraic expressions is merely the abstract manipulation of symbols. Use of real-world context examples to demonstrate the meaning of the parts of algebraic expressions is needed to counter this misconception.

Students may also believe that an expression cannot be factored because it does not fit into a form they recognize. They need help with reorganizing the terms until structures become evident.

Some students may believe that factoring and completing the square are isolated techniques within a unit of quadratic equations. Teachers should help students to see the value of these skills in the context of solving higher degree equations and examining different families of functions.

- **I can solve quadratic equations by factoring.**

(From Ohio's Model Curricula) This cluster focuses on linking expressions and functions, i.e., creating connections between multiple representations of functional relations – the dependence between a quadratic expression and a graph of the quadratic function it defines, and the dependence between different symbolic representations of exponential functions. Teachers need to foster the idea that changing the forms of expressions, such as factoring or completing the square, or transforming expressions from one exponential form to another, are not independent algorithms that are learned for the sake of symbol manipulations. They are processes that are guided by goals (e.g., investigating properties of families of functions and solving contextual problems).

Factoring methods that are typically introduced in elementary algebra and the method of completing the square reveals attributes of the graphs of quadratic functions, represented by quadratic equations.

- The solutions of quadratic equations solved by factoring are the x – intercepts of the parabola or zeroes of quadratic functions.

- **I can sketch a rough graph using the zeroes of a quadratic function and other easily identifiable points.**

(From Ohio's Model Curricula) Factoring methods that are typically introduced in elementary algebra and the method of completing the square reveals attributes of the graphs of quadratic functions, represented by quadratic equations.

- The solutions of quadratic equations solved by factoring are the x – intercepts of the parabola or zeroes of quadratic functions.
- A pair of coordinates (h, k) from the general form $f(x) = a(x – h)^2 + k$ represents the vertex of the parabola, where h represents a horizontal shift and k represents a vertical shift of the parabola $y = x^2$ from its original position at the origin.
- A vertex (h, k) is the minimum point of the graph of the quadratic function if a › 0 and is the maximum point of the graph of the quadratic function if a ‹ 0. Understanding an algorithm of completing the square provides a solid foundation for deriving a quadratic formula.

Translating among different forms of expressions, equations and graphs helps students to understand some key connections among arithmetic, algebra and geometry. The reverse thinking technique (a process that allows working backwards from the answer to the starting point) can be very effective. Have students derive information about a function's equation, represented in standard, factored or general form, by investigating its graph.

Before using graphing technology to explore the effect of changing constants in quadratic equations, allow students to first make tables by hand.

Common Misconceptions from Ohio's Model Curricula:
Students may think that the minimum (the vertex) of the graph of $y = (x + 5)^2$ is shifted to the right of the minimum (the vertex) of the graph $y = x^2$ due to the addition sign. Students should explore examples both analytically and graphically to overcome this misconception.

Some students may believe that the minimum of the graph of a quadratic function always occur at the *y*-intercept.

Literacy Standards Considerations:

- **I can add and subtract polynomials.**

Students would need to be familiar with the definitions of key terms.

- **I can multiply polynomials.**

Students would need to be familiar with the definitions of key terms.

- **I can rewrite an expression using factoring.**

Students would need to be familiar with the definitions of key terms.

- **I can solve quadratic equations by factoring.**

Word Problems are included as a part of the practice set - students have to interpret into algebraic notation.

- **I can sketch a rough graph using the zeroes of a quadratic function and other easily identifiable points.**

Students would need to be familiar with the definitions of key terms.

Strategies for Diverse Learners:
Problems of varying difficulty are incorporated into the problem sets.

Instructional Resources:
Holt McDougal Algebra 1 © 2011

- **I can add and subtract polynomials.**

- **I can multiply polynomials.**

- **I can rewrite an expression using factoring.**

http://www.openmiddle.com/factoring-quadratics/

- **I can solve quadratic equations by factoring.**

http://squarerootofnegativeoneteachmath.blogspot.com/2013/09/this-lesson-cost-me-1.html

- **I can sketch a rough graph using the zeroes of a quadratic function and other easily identifiable points.**

Assessment:
Formative:
- exit slips
- observation of students working on problems in class
- observation of student work from outside of class practice problems

Summative:
- unit test questions

Assessment Resources:
http://www.illustrativemathematics.org/
http://practice.parcc.testnav.com/
IIS
Holt McDougal Algebra 1 © 2011

Unit 8 – Quadratic Functions

(3 weeks, March)

<u>Common Core State Standards Addressed:</u>

A-APR.3: Identify zeroes of polynomials when suitable factorizations are available, and use the zeroes to construct a rough graph of the function defined by the polynomial.
A-SSE.3a: Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.* Factor a quadratic expression to reveal the zeroes of the function it defines.
A-SSE.3b: Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.* Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
F-IF.4: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity*.*
F-IF.7a: Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.* Graph linear and quadratic functions and show intercepts, maxima, and minima.
F-IF.8a: Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the process of factoring and completing the square in a quadratic function to show zeroes, extreme values, and symmetry of the graph, and interpret these in terms of a context.
F-BF.3: Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
S-ID.6a: Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.

<u>Student Friendly Learning Targets (In order of teaching):</u>

- I can use completing the square to rewrite a quadratic expression into vertex form.
- I can graph a quadratic function, identifying key features such as the intercepts, maximum and/or minimum values, symmetry, and end behavior of the graph.
- I can identify the effect of transformations on the graph of a function with and without technology.
- I can construct a scatter plot, use technology to find a quadratic function of best fit, and use that function to make predictions.

<u>Vocabulary:</u>
quadratic expression, quadratic equation, zeroes, perfect-square trinomial, complete the square, function, maximum, minimum, x-intercept, y-intercept, interval, increase, decrease, symmetry, end behavior, evaluate, domain, range, input, output, parent function, transformation, coordinate plane, vertex, quadratic function, turning point, factor, vertex form, axis of symmetry, translate, stretch, shrink, scatter plot, quantitative variable, independent variable, dependent variable, scale, direction, form, strength, quadratic model, function of best fit

<u>Materials and/or Technology Needed:</u>
calculator, graphing calculator, Desmos.com, graph paper

<u>Instructional Strategies:</u>
Information from all Learning Targets can be found at Ohio's Model Curricula: http://education.ohio.gov/Topics/Academic-Content-Standards/Mathematics and KCTM's Flip Book (which encompasses several states' information including Ohio): http://katm.org/wp/wp-content/uploads/flipbooks/High-School-CCSS-Flip-Book-USD-259-2012.pdf

- **I can use completing the square to rewrite a quadratic expression into vertex form.**

(From Ohio's Model Curricula) This cluster focuses on linking expressions and functions, i.e., creating connections between multiple representations of functional relations – the dependence between a quadratic expression and a graph of the quadratic function it defines, and the dependence between different symbolic representations of exponential functions. Teachers need to foster the idea that changing the forms of expressions, such as factoring or completing the square, or transforming expressions from one exponential form to another, are not independent algorithms that are learned for the sake of symbol manipulations. They are processes that are guided by goals (e.g., investigating properties of families of functions and solving contextual problems).

Factoring methods that are typically introduced in elementary algebra and the method of completing the square reveals attributes of the graphs of quadratic functions, represented by quadratic equations.

<u>Common Misconceptions from Ohio's Model Curricula:</u>
Some students may believe that factoring and completing the square are isolated techniques within a unit of quadratic equations. Teachers should help students to see the value of these skills in the context of solving higher degree equations and examining different families of functions.

- **I can graph a quadratic function, identifying key features such as the intercepts, maximum and/or minimum values, symmetry, and end behavior of the graph.**

(From Ohio's Model Curricula) Factoring methods that are typically introduced in elementary algebra and the method of completing the square reveals attributes of the graphs of quadratic functions, represented by quadratic equations.

- The solutions of quadratic equations solved by factoring are the *x* – intercepts of the parabola or zeroes of quadratic functions.
- A pair of coordinates (*h*, *k*) from the general form $f(x) = a(x – h)^2 + k$ represents the vertex of the parabola, where *h* represents a horizontal shift and k represents a vertical shift of the parabola $y = x^2$ from its original position at the origin.
- A vertex (*h, k*) is the minimum point of the graph of the quadratic function if a › 0 and is the maximum point of the graph of the quadratic function if a ‹ 0. Understanding an algorithm of completing the square provides a solid foundation for deriving a quadratic formula.

Translating among different forms of expressions, equations and graphs helps students to understand some key connections among arithmetic, algebra and geometry. The reverse thinking technique (a process that allows working backwards from the answer to the starting point) can be very effective. Have students derive information about a function’s equation, represented in standard, factored or general form, by investigating its graph.

Flexibly move from examining a graph and describing its characteristics (e.g., intercepts, relative maximums, etc.) to using a set of given characteristics to sketch the graph of a function.

Examine a table of related quantities and identify features in the table, such as intervals on which the function increases, decreases, or exhibits periodic behavior.

Discover that the factored form of a quadratic or polynomial equation can be used to determine the zeroes, which in turn can be used to identify maxima, minima and end behaviors.

Use various representations of the same function to emphasize different characteristics of that function. For example, the *y*-intercept of the function $y = x^2 - 4x – 12$ is easy to recognize as (0, -12). However, rewriting the function as $y = (x – 6)(x + 2)$ reveals zeroes at (6, 0) and at (-2, 0). Furthermore, completing the square allows the equation to be written as $y = (x – 2)^2 – 16$, which shows that the vertex (and minimum point) of the parabola is at (2, -16).

<u>Common Misconceptions from Ohio’s Model Curricula:</u>
Students may think that the minimum (the vertex) of the graph of $y = (x + 5)^2$ is shifted to the right of the minimum (the vertex) of the graph $y = x^2$ due to the addition sign. Students should explore examples both analytically and graphically to overcome this misconception.

Some students may believe that the minimum of the graph of a quadratic function always occur at the *y*-intercept.

Students may also believe that skills such as factoring a trinomial or completing the square are isolated within a unit on polynomials, and that they will come to understand the usefulness of these skills in the context of examining characteristics of functions.

Additionally, student may believe that the process of rewriting equations into various forms is simply an algebra symbol manipulation exercise, rather than serving a purpose of allowing different features of the function to be exhibited.

- **I can identify the effect of transformations on the graph of a function with and without technology.**

(From Ohio's Model Curricula) Translating among different forms of expressions, equations and graphs helps students to understand some key connections among arithmetic, algebra and geometry. The reverse thinking technique (a process that allows working backwards from the answer to the starting point) can be very effective. Have students derive information about a function's equation, represented in standard, factored or general form, by investigating its graph.

Explore various families of functions and help students to make connections in terms of general features. For example, just as the function $y = (x + 3)^2 – 5$ represents a translation of the function $y = x$ by 3 units to the left and 5 units down, the same is true for the function $y = | x + 3 | - 5$ as a translation of the absolute value function $y = | x |$.

<u>Common Misconceptions from Ohio's Model Curricula:</u>
<u>Students may believe</u> that each family of functions (e.g., quadratic, square root, etc.) is independent of the others, so they may not recognize commonalities among all functions and their graphs.

Students may also believe that skills such as factoring a trinomial or completing the square are isolated within a unit on polynomials, and that they will come to understand the usefulness of these skills in the context of examining characteristics of functions.

Additionally, student may believe that the process of rewriting equations into various forms is simply an algebra symbol manipulation exercise, rather than serving a purpose of allowing different features of the function to be exhibited.

- **I can construct a scatter plot, use technology to find a quadratic function of best fit, and use that function to make predictions.**

(From Ohio's Model Curricula) In the numerical or quantitative case, display the paired data in a scatterplot. Note that although the two variables in general will not have the same scale, e.g., total SAT versus grade-point average, it is best to begin with variables with the same scale such as SAT Verbal and SAT Math. Fitting functions to such data will avoid difficulties such as interpretation of slope in the linear case in which scales differ. Once students are comfortable with the same scale case, introducing different scales situations will be less problematic.

<u>**Strategies for Diverse Learners:**</u>
Problems of varying difficulty are incorporated into the problem sets.

Literacy Standards Considerations:

- **I can use completing the square to rewrite a quadratic expression into vertex form.**

Students would need to be familiar with the definitions of key terms.

- **I can graph a quadratic function, identifying key features such as the intercepts, maximum and/or minimum values, symmetry, and end behavior of the graph.**

Students would need to be familiar with the definitions of key terms.

- **I can identify the effect of transformations on the graph of a function with and without technology.**

Students would need to be familiar with the definitions of key terms.

- **I can construct a scatter plot, use technology to find a quadratic function of best fit, and use that function to make predictions.**

Students would need to be familiar with the definitions of key terms. Word Problems are included as a part of the practice set - students will have to interpret the key terms as they apply to the problem.

Instructional Resources:

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- **I can use completing the square to rewrite a quadratic expression into vertex form.**

https://docs.google.com/file/d/0B_6HGo8g8V7jaG1PZXVEakF1VzA/edit
http://function-of-time.blogspot.com/2011/11/completing-square.html
http://untilnextstop.blogspot.com/2012/05/completing-square-geometrically.html
http://mrsreillyblog.wordpress.com/2013/03/17/completing-the-square-activity/

- **I can graph a quadratic function, identifying key features such as the intercepts, maximum and/or minimum values, symmetry, and end behavior of the graph.**

http://mrshester.blogspot.com/2013/07/properties-of-quadratic-graphs.html
http://fawnnguyen.com/2013/03/20/des-man.aspx

- **I can identify the effect of transformations on the graph of a function with and without technology.**

http://reflectionsfromanasymptote.wordpress.com/2014/02/14/new-activity-algebraic-transformations/
http://partiallyderivative.wordpress.com/2013/12/04/function-transformations-card-matching-activity/

- **I can construct a scatter plot, use technology to find a quadratic function of best fit, and use that function to make predictions.**

Assessment:

Formative:

- exit slips
- observation of students working on problems in class
- observation of student work from outside of class practice problems

Summative:
- unit test questions

Assessment Resources:
http://www.illustrativemathematics.org/
http://practice.parcc.testnav.com/
IIS
Holt McDougal Algebra 1 © 2011

Unit 9 – Quadratic Equations

(2 ½ weeks, April)

Common Core State Standards Addressed:

N-RN.3: Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

A-REI.4a: Solve quadratic equations in one variable. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.

A-REI.4b: Solve quadratic equations in one variable. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

Student Friendly Learning Targets (In order of teaching):

- I can explain why sums and products are either rational or irrational.
- I can solve quadratic equations by completing the square.
- I can solve quadratic equations by finding square roots.
- I can solve quadratic equations by using the quadratic formula.

Vocabulary:

real number, rational number, irrational number, sum, product, quadratic equation, complete the square, inspection, square root method, quadratic formula, complex solution, factoring completely, radicand, imaginary number, perfect square trinomial

Materials and/or Technology Needed:

calculator

Instructional Strategies:

Information from all Learning Targets can be found at Ohio's Model Curricula: http://education.ohio.gov/Topics/Academic-Content-Standards/Mathematics and KCTM's Flip Book (which encompasses several states' information including Ohio): http://katm.org/wp/wp-content/uploads/flipbooks/High-School-CCSS-Flip-Book-USD-259-2012.pdf

- **I can explain why sums and products are either rational or irrational.**

(From Ohio Model Curricula) This cluster is an excellent opportunity to incorporate algebraic proof, both direct and indirect, in teaching properties of number systems.

Students should explore concrete examples that illustrate that for any two rational numbers written in form a/b and c/d, where b and d are natural numbers and a and c are integers, the following are true:

$\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$ represents a rational number, and $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$ represents a rational number.

Continue exploring situations where the sum of a rational number and an irrational number is irrational (e.g., a sum of rational number 2 and irrational number √3 is (2 + √3), which is an irrational).

Proofs are valid ways to justify not only geometry statements also algebraic statements.
Use indirect algebraic proof to generalize the statement that the sum of a rational and irrational number is irrational.

Assume that *x* is an irrational number and the sum of x and a rational number $\frac{a}{b}$ is also rational and is represented as $\frac{c}{d}$

$x + \frac{a}{b} = \frac{c}{d}$

$x = \frac{c}{d} - \frac{a}{b}$

$x = \frac{cb-ad}{bd}$ represents a rational number

Since the last statement contradicts a given fact that *x* is an irrational number, the assumption is wrong and a sum of a rational number and an irrational number has to be irrational. Similarly, it can be proven that the product of a non- zero rational and an irrational number is irrational.

Students need to see that results of the operations performed between numbers from a particular number set does not always belong to the same set. For example, the sum of two irrational numbers$(2 + \sqrt{3})$ and $(2 - \sqrt{3})$ is 4, which is a rational number.

<u>Common Misconceptions (from Ohio Model Curricula):</u>
Some students may believe that both terminating and repeating decimals are rational numbers, without considering nonrepeating and nonterminating decimals as irrational numbers.

Students may also confuse irrational numbers and complex numbers, and therefore mix their properties. In this case, students should encounter examples that support or contradict properties and relationships between number sets (i.e., irrational numbers are real numbers and complex numbers are non-real numbers. The set of real numbers is a subset of the set of complex numbers).

By using false extensions of properties of rational numbers, some students may assume that the sum of any two irrational numbers is also irrational. This statement is not always true (e.g., $(2 + \sqrt{3}) + (2 - \sqrt{3}) = 4$, a rational number), and therefore, cannot be considered as a property.

- **I can solve quadratic equations by completing the square.**
- **I can solve quadratic equations by finding square roots.**
- **I can solve quadratic equations by using the quadratic formula.**

(For all three from Ohio's Model Curricula) Completing the square is usually introduced for several reasons" to find the vertex of a parabola whose equation has been expanded; to look at the parabola through the lenses of translations of a "parent" parabola $y = x^2$; and to derive a quadratic formula. Completing the square is a very useful tool that will be used repeatedly by students in many areas of mathematics. Teachers should carefully balance traditional paper-pencil skills of manipulating quadratic expressions and solving quadratic equations along with an analysis of the relationship between parameters of quadratic equations and properties of their graphs.

Start by inspecting equations such as $x^2 = 9$ that has two solutions, 3 and -3. Next, progress to equations such as $(x - 7)^2 = 9$ by substituting $x – 7$ for x and solving them either by "inspection" or by taking the square root on each side:

$$x - 7 = 3 \quad \text{and} \quad x - 7 = -3$$
$$x = 10 \qquad\qquad x = 4$$

Graph both pairs of solutions (-3 and 3, 4 and 10) on the number line and notice that 4 and 10 are 7 units to the right of – 3 and 3. So, the substitution of $x – 7$ for x moved the solutions 7 units to the right. Next, graph the function $y = (x – 7)^2 – 9$, pointing out that the x-intercepts are 4 and 10, and emphasizing that the graph is the translation of 7 units to the right and 9 units down from the position of the graph of the parent function $y = x^2$ that passes through the origin (0, 0). Generate more equations of the form $y = a(x – h)^2 + k$ and compare their graphs using a graphing technology.

Highlight and compare different approaches to solving the same problem. Use technology to recognize that two different expressions or equations may represent the same relationship. For example, since $x^2 -10x +25 = 0$ can be rewritten as $(x- 5)(x –5) = 0$ or $(x – 5)^2 = 0$ or $x^2 = 25$, these are all representations of the same equation that has a double solution x = 5. Support it by putting all expressions into graphing calculator. Compare their graphs and generate their tables displaying the same output values for each expression.

Guide students in transforming a quadratic equation in standard form, $0 = ax^2 + bx + c$, to the vertex form $0 = a(x – h)^2 + k$ by separating your examples into groups with $a = 1$ and $a \neq 1$ and have students guess the number that needs to be added to the binomials of the type $x^2 + 6x$, $x^2 – 2x$, $x^2 +9x$, $x^2 - \frac{2}{3}x$ to form complete square of the binomial $(x – m)^2$. Then generalize the process by showing the expression $(b/2)^2$ that has to be added to the binomial $x^2 + bx$. Completing the square for an expression whose x^2 coefficient is not 1 can be complicated for some students. Present multiple examples of the type $0 = 2x^2 – 5x - 9$ to emphasize the logic behind every step, keeping in mind that the same process will be used to complete the square in the proof of the quadratic formula.

Discourage students from giving a preference to a particular method of solving quadratic equations. Students need experience in analyzing a given problem to choose an appropriate solution method before their computations become burdensome. Point out that the Quadratic

Formula, x = $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$, is a universal tool that can solve any quadratic equation; however, it is not reasonable to use the Quadratic Formula when the quadratic equation is missing either a middle term, bx, or a constant term, c. When it is missing a constant term, (e.g., $3x^2 - 10x = 0$) a factoring method becomes more efficient. If a middle term is missing (e.g., $2x^2 - 15 = 0$), a square root method is the most appropriate. Stress the benefit of memorizing the Quadratic Formula and flexibility with a factoring strategy. Introduce the concept of discriminants and their relationship to the number and nature of the roots of quadratic equation.

, Offer students examples of a quadratic equation, such as $x^2 + 9 = 0$. Since the graph of the quadratic function $y = x^2 + 9$ is situated above the x –axis and opens up, the graph does not have x–intercepts and therefore, the quadratic equation does not have real solutions. At this stage introduce students to non-real solutions, such as $x = \pm\sqrt{-9}$ or $x = \pm 3\sqrt{-1}$ and a new number type-imaginary unit *i* that equals $\sqrt{-1}$. Using *i* in place of $\sqrt{-1}$ is a way to present the two solutions of a quadratic equation in the complex numbers form $a \pm bi$, if *a* and *b* are real numbers and $b \neq 0$. Have students observe that if a quadratic equation has complex solutions, the solutions always appear in conjugate pairs, in the form $a + bi$ and $a - bi$. Particularly, for the equation $x^2 = -9$, a conjugate pair of solutions are $0 + 3i$ and
$0 - 3i$. Project the same logic in the examples of any quadratic equations $0 = ax^2 + bx + c$ that have negative discriminants. The solutions are a pair of conjugate complex numbers $\frac{-b\pm i\sqrt{D}}{2a}$, if D = $b^2 - 4ac$ is negative.

<u>Common Misconceptions from Ohio's Model Curricula:</u>
Some students may think that rewriting equations into various forms (taking square roots, completing the square, using quadratic formula and factoring) are isolated techniques within a unit of quadratic equations. Teachers should help students see the value of these skills in the context of solving higher degree equations and examining different families of functions.

<u>Strategies for Diverse Learners:</u>
Problems of varying difficulty are incorporated into the problem sets.

<u>Literacy Standards Considerations:</u>

- **I can explain why sums and products are either rational or irrational.**

Students would need to be familiar with the definitions of key terms.

- **I can solve quadratic equations by completing the square.**

Word Problems are included as a part of the practice set - students have to interpret into algebraic notation.

- **I can solve quadratic equations by finding square roots.**

Word Problems are included as a part of the practice set - students have to interpret into algebraic notation.

- **I can solve quadratic equations by using the quadratic formula.**

Word Problems are included as a part of the practice set - students have to interpret into algebraic notation.

Instructional Resources:

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- **I can explain why sums and products are either rational or irrational.**

- **I can solve quadratic equations by completing the square.**

http://untilnextstop.blogspot.com/2012/05/completing-square-geometrically.html
http://map.mathshell.org/materials/mapbeta/lessons.php?taskid=432#task432
http://mrsreillyblog.wordpress.com/2013/03/17/completing-the-square-activity/
http://function-of-time.blogspot.com/2011/11/completing-square.html

- **I can solve quadratic equations by finding square roots.**

- **I can solve quadratic equations by using the quadratic formula.**

Assessment:

Formative:

- exit slips
- observation of students working on problems in class
- observation of student work from outside of class practice problems

Summative:

- unit test questions

Assessment Resources:

http://www.illustrativemathematics.org/
http://practice.parcc.testnav.com/
IIS
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Unit 10 – Relationships That Are Not Linear

(2 weeks, April)

Common Core State Standards Addressed:

N-Q.1: Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
N-RN.1: Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.*
N-RN.2: Rewrite expressions involving radicals and rational exponents using the properties of exponents.
F-IF.7b: Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.* Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

Student Friendly Learning Targets (In order of teaching):

- I can apply the properties of exponents to simplify algebraic expressions with rational exponents.
- I can graph a square root or cube root function, identifying key features such as the intercepts, maximum and/or minimum values, and end behavior of the graph.
- I can graph a piecewise function, including step and absolute value functions, identifying key features such as the intercepts, maximum and/or minimum values, and end behavior of the graph.

Vocabulary:

exponent, laws of exponents, simplify, expression, integer, rational, evaluate, function, domain, range, input, output, x-intercept, y-intercept, coordinate plane, maximum, minimum, square root function, cube root function, piecewise function, step function, absolute value function, end behavior

Materials and/or Technology Needed:

calculator, graphing calculator, Desmos.com, graph paper

Instructional Strategies:

Information from all Learning Targets can be found at Ohio's Model Curricula: http://education.ohio.gov/Topics/Academic-Content-Standards/Mathematics and KCTM's Flip Book (which encompasses several states' information including Ohio): http://katm.org/wp/wp-content/uploads/flipbooks/High-School-CCSS-Flip-Book-USD-259-2012.pdf

For all learning targets involving graphing, from the Ohio Model Curricula:

Graphical representations serve as visual models for understanding phenomena that take place in our daily surroundings. The use of different kinds of graphical representations along with their units, labels and titles demonstrate the level of students' understanding and foster the ability to reason, prove, self-check, examine relationships and establish the validity of arguments. Students need to be able to identify misleading graphs by choosing correct units and scales to create a correct representation of a situation or to make a correct conclusion from it.

- **I can apply the properties of exponents to simplify algebraic expressions with rational exponents.**

(From Ohio Model Curricula) The goal is to show that a **fractional exponent can be expressed as a radical or a root**. For example, an exponent of 1/3 is equivalent to a cube root; an exponent of ¼ is equivalent to a fourth root.

Review the power rule, $((b^n)^m = b^{nm})$, for whole number exponents (e.g. $(7^2)^3 = 7^6$.

Compare examples, such as $(7^{1/2})^2 = 7^1 = 7$ and $(\sqrt{7})^2 = 7$, to help students establish a connection between radicals and rational exponents: $7^{\frac{1}{2}} = \sqrt{7}$ and, in general, $b^{\frac{1}{2}} = \sqrt{b}$.

Provide opportunities for students to explore the equality of the values using calculators, such as $7^{1/2}$ and $\sqrt{7}$. Offer sufficient examples and exercises to prompt the definition of fractional exponents, and give students practice in converting expressions between radical and exponential forms.

When n is a positive integer, generalize the meaning of $b^{\frac{1}{n}} = \sqrt[n]{b^1}$ and then to $b^{\frac{m}{n}} = \sqrt[n]{b^m}$, where n and m are integers and n is greater than or equal to 2. When m is a negative integer, the result is the reciprocal of the root $b^{\frac{-m}{n}} = \frac{1}{\sqrt[n]{b^m}}$.

Stress the two rules of rational exponents: 1) the numerator of the exponent is the base's power and 2) the denominator of the exponent is the order of the root. When evaluating expressions involving rational exponents, it is often helpful to break an exponent into its parts – a power and a root – and then decide if it is easier to perform the root operation or the exponential operation first.

Model the use of precise mathematics vocabulary (e.g., base, exponent, radical, root, cube root, square root etc.).

The rules for integer exponents are applicable to rational exponents as well; however, the operations can be slightly more complicated because of the fractions. When multiplying exponents, powers are added$(b^n \cdot b^m = b^{n+m})$. When dividing exponents, powers are subtracted $(\frac{b^n}{b^m} = b^{n-m})$. When raising an exponent to an exponent, powers are multiplied $((b^n)^m = b^{nm})$.

<u>Common Misconceptions (From Ohio Model Curricula):</u>
Students sometimes misunderstand the meaning of exponential operations, the way powers and roots relate to one another, and the order in which they should be performed. Attention to the base is very important. Consider examples: $\left(-81^{\frac{3}{4}}\right)$ and $(-81)^{\frac{3}{4}}$. The position of a negative sign of a term with a rational exponent can mean that the rational exponent should be either applied first to the base, 81, and then the opposite of the result is taken $\left(-81^{\frac{3}{4}}\right)$, or the rational exponent should be applied to a negative term $(-81)^{\frac{3}{4}}$. The answer of $\sqrt[4]{-81}$ will be not real if the denominator of the exponent is even. If the root is odd, the answer will be a negative number.

Students should be able to make use of estimation when incorrectly using multiplication instead of exponentiation.

Students may believe that the fractional exponent in the expression $36^{\frac{1}{3}}$ means the same as a factor $\frac{1}{3}$ in multiplication expression, $36 \cdot \frac{1}{3}$ and multiply the base by the exponent.

- **I can graph a square root or cube root function, identifying key features such as the intercepts, maximum and/or minimum values, and end behavior of the graph.**
- **I can graph a piecewise function, including step and absolute value functions, identifying key features such as the intercepts, maximum and/or minimum values, and end behavior of the graph.**

(For both from Ohio's Model Curricula) Explore various families of functions and help students to make connections in terms of general features. For example, just as the function $y = (x + 3)^2 - 5$ represents a translation of the function $y = x$ by 3 units to the left and 5 units down, the same is true for the function $y = | x + 3 | - 5$ as a translation of the absolute value function $y = | x |$.

<u>Common Misconceptions from Ohio's Model Curricula:</u>
<u>Students may believe</u> that each family of functions (e.g., quadratic, square root, etc.) is independent of the others, so they may not recognize commonalities among all functions and their graphs.

<u>Strategies for Diverse Learners:</u>
Problems of varying difficulty are incorporated into the problem sets.

<u>Literacy Standards Considerations:</u>
- **I can apply the properties of exponents to simplify algebraic expressions with rational exponents.**

Students would need to be familiar with the definitions of key terms.

- **I can graph a square root or cube root function, identifying key features such as the intercepts, maximum and/or minimum values, and end behavior of the graph.**

Students would need to be familiar with the definitions of key terms.

- **I can graph a piecewise function, including step and absolute value functions, identifying key features such as the intercepts, maximum and/or minimum values, and end behavior of the graph.**

Students would need to be familiar with the definitions of key terms.

Instructional Resources:

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- **I can apply the properties of exponents to simplify algebraic expressions with rational exponents.**

- **I can graph a square root or cube root function, identifying key features such as the intercepts, maximum and/or minimum values, and end behavior of the graph.**

- **I can graph a piecewise function, including step and absolute value functions, identifying key features such as the intercepts, maximum and/or minimum values, and end behavior of the graph.**

http://mathteachermambo.blogspot.com/2013/09/piecewise-functions.html

Assessment:

Formative:

- exit slips
- observation of students working on problems in class
- observation of student work from outside of class practice problems

Summative:

- unit test questions

Assessment Resources:

http://www.illustrativemathematics.org/
http://practice.parcc.testnav.com/
IIS
Holt McDougal Algebra 1 © 2011

Unit 11 – Exponential Functions and Equations

(4 weeks, April, May)

Common Core State Standards Addressed:

A-SSE.1b: Interpret expressions that represent a quantity in terms of its context. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.*

A-SSE.3c: Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.* Use the properties of exponents to transform expressions for exponential functions. *For example the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*

F-IF.7e: Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.* Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

F-LE.1a: Distinguish between situations that can be modeled with linear functions and with exponential functions. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.

F-LE.1c: Distinguish between situations that can be modeled with linear functions and with exponential functions. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval to another.

F-LE.3: Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

F-LE.5: Interpret the parameters in a linear or exponential function in terms of a context.

S-ID.6a: Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.

Student Friendly Learning Targets (In order of teaching):

- I can demonstrate that an exponential function has a constant multiplier over equal intervals.
- I can identify situations that display equal ratios of change over equal intervals and can be modeled with exponential functions.
- I can use graphs or tables to compare the rates of change of linear, quadratic, and exponential functions.
- I can rewrite exponential functions using the properties of exponents.
- I can interpret the parameters of an exponential function in real-life problems.
- I can graph exponential functions, identifying key features such as the intercepts, maximum and/or minimum values, asymptotes, and end behavior of the graph.

- I can construct a scatter plot, use technology to find an exponential function of best fit, and use that function to make predictions.

Vocabulary:
function, maximum, minimum, exponential function, evaluate, domain, range, input, output, equation, x-intercept, coordinate plane, end behavior, asymptote, exponential function, common ratio, rate, growth rate, scatter plot, quantitative variable, independent variable, dependent variable, scale, direction, form, exponential model, function of best fit, data set

Materials and/or Technology Needed:
calculator, graphing calculator, Desmos.com, Excel, graph paper

Instructional Strategies:
Information from all Learning Targets can be found at Ohio's Model Curricula: http://education.ohio.gov/Topics/Academic-Content-Standards/Mathematics and KCTM's Flip Book (which encompasses several states' information including Ohio): http://katm.org/wp/wp-content/uploads/flipbooks/High-School-CCSS-Flip-Book-USD-259-2012.pdf

- **I can demonstrate that an exponential function has a constant multiplier over equal intervals.**
- **I can identify situations that display equal ratios of change over equal intervals and can be modeled with exponential functions.**

(For both from Ohio's Model Curricula) Compare tabular representations of a variety of functions to show that linear functions have a first common difference (i.e., equal differences over equal intervals), while exponential functions do not (instead function values grow by equal factors over equal x-intervals).

Apply linear and exponential functions to real-world situations. For example, a person earning $10 per hour experiences a constant rate of change in salary given the number of hours worked, while the number of bacteria on a dish that doubles every hour will have equal factors over equal intervals.

Common Misconceptions from Ohio's Model Curricula:
Students may believe that all functions have a first common difference and need to explore to realize that, for example, a quadratic function will have equal second common differences in a table.

- **I can use graphs or tables to compare the rates of change of linear, quadratic, and exponential functions.**

(From Ohio's Model Curricula) Use a graphing calculator or computer program to compare tabular and graphic representations of exponential and polynomial functions to show how the y (output) values of the exponential function eventually exceed those of polynomial functions.

Before using the formula, it might be reasonable to demonstrate the way the formula is derived,

Have students draw the graphs of exponential and other polynomial functions on a graphing calculator or computer utility and examine the fact that the exponential curve will eventually get higher than the polynomial function's graph. A simple example would be to compare the graphs (and tables) of the functions $y = x^2$ and $y = 2^x$ to find that the *y* values are greater for the exponential function when $x > 4$.

<u>Common Misconceptions from Ohio's Model Curricula:</u>
Students may believe that all functions have a first common difference and need to explore to realize that, for example, a quadratic function will have equal second common differences in a table.

Students may also believe that the end behavior of all functions depends on the situation and not the fact that exponential function values will eventually get larger than those of any other polynomial functions.

- **I can rewrite exponential functions using the properties of exponents.**

(From Ohio's Model Curricula) Translating among different forms of expressions, equations and graphs helps students to understand some key connections among arithmetic, algebra and geometry. The reverse thinking technique (a process that allows working backwards from the answer to the starting point) can be very effective. Have students derive information about a function's equation, represented in standard, factored or general form, by investigating its graph.

Offer multiple real-world examples of exponential functions. For instance, to illustrate an exponential decay, students need to recognize that in the equation for an automobile cost $C(t) = 20{,}000(0.75)^t$, the base is 0.75 and between 0 and 1 and the value of $20,000 represents the initial cost of an automobile that depreciates 25% per year over the course of *t* years.

Similarly, to illustrate exponential growth, in the equation for the value of an investment over time $A(t) = 10{,}000(1.03)^t$, where the base is 1.03 and is greater than 1; and the $10,000 represents the value of an investment when increasing in value by 3% per year for *x* years.

A problem such as, "An amount of $100 was deposited in a savings account on January 1st each of the years 2010, 2011, 2012, and so on to 2019, with annual yield of 7%. What will be the balance in the savings account on January 1, 2020?" illustrates the use of a formula for a geometric series $\boldsymbol{S_n = \frac{g(1-r^n)}{(1-r)}}$ when S_n represents the value of the geometric series with the first term g, constant ration $r \neq 1$, and n terms.

$$S_n = g + gr + gr^2 + gr^3 + \dots gr^{n-1}.$$

Multiply by r $rS_n = gr + gr^2 + \ldots + gr^{n-1} + gr^n$

Subtract $S - rS = g - gr^n$

Factor $S(1 - r) = g(1 - r^n)$

Divide by $(1 - r)$ $\boldsymbol{S_n = \frac{g(1-r^n)}{(1-r)}}$

The amount of the investment for January 1, 2020 can be found using: $100(1.07)^{10} + 100(1.07)^9 + \ldots + 100(1.07)$. If the first term of this geometric series is $g = 100(1.07)$, the ratio is 1.07 and the number of terms $n = 10$, the formula for the value of geometric series is:

$$\boldsymbol{S_{10} = \frac{g(1-r^{10})}{(1-r)} = \frac{100(1.07)(1.07^{10}-1)}{(1.07-1)} = \frac{107(1.07^{10}-1)}{0.07}}$$

$S_{10} \approx \$1478.36$

- **I can interpret the parameters of an exponential function in real-life problems.**

(From Ohio's Model Curricula) Use real-world contexts to help students understand how the parameters of linear and exponential functions depend on the context. For example, a plumber who charges \$50 for a house call and \$85 per hour would be expressed as the function $y = 85x + 50$, and if the rate were raised to \$90 per hour, the function would become $y = 90x + 50$. On the other hand, an equation of $y = 8{,}000(1.04)^x$ could model the rising population of a city with 8,000 residents when the annual growth rate is 4%. Students can examine what would happen to the population over 25 years if the rate were 6% instead of 4% or the effect on the equation and function of the city's population were 12,000 instead of 8,000.

Graphs and tables can be used to examine the behaviors of functions as parameters are changed, including the comparison of two functions such as what would happen to a population if it grew by 500 people per year, versus rising an average of 8% per year over the course of 10 years.

<u>Common Misconceptions from Ohio's Model Curricula:</u>
Students may believe that changing the slope of a linear function from "2" to "3" makes the graph steeper without realizing that there is a real-world context and reason for examining the slopes of lines. Similarly, an exponential function can appear to be abstract until applying it to a real-world situation involving population, cost, investments, etc.

- **I can graph exponential functions, identifying key features such as the intercepts, maximum and/or minimum values, asymptotes, and end behavior of the graph.**

(From Ohio's Model Curricula) Examine multiple real-world examples of exponential functions so that students recognize that a base between 0 and 1 (such as an equation describing depreciation of an automobile [$f(x) = 15{,}000(0.8)^x$ represents the value of a \$15,000 automobile that depreciates 20% per year over the course of x years]) results in an exponential decay, while a base greater than 1 (such as the value of an investment over time [$f(x) = 5{,}000(1.07)^x$ represents the value of an investment of $5,000 when increasing in value by 7% per year for *x* years]) illustrates growth.

<u>Common Misconceptions from Ohio's Model Curricula:</u>
<u>Students may believe</u> that each family of functions (e.g., quadratic, square root, etc.) is independent of the others, so they may not recognize commonalities among all functions and their graphs.

- **I can construct a scatter plot, use technology to find an exponential function of best fit, and use that function to make predictions.**

(From Ohio's Model Curricula) In the numerical or quantitative case, display the paired data in a scatterplot. Note that although the two variables in general will not have the same scale, e.g., total SAT versus grade-point average, it is best to begin with variables with the same scale such as SAT Verbal and SAT Math. Fitting functions to such data will avoid difficulties such as interpretation of slope in the linear case in which scales differ. Once students are comfortable with the same scale case, introducing different scales situations will be less problematic.

(From Ohio's Model Curricula) Extending beyond simplifying an expression, this cluster addresses interpretation of the components in an algebraic expression. A student should recognize that in the expression $2x + 1$, "2" is the coefficient, "2" and "*x*" are factors, and "1" is a constant, as well as "2*x*" and "1" being *terms* of the binomial expression. Development and proper use of mathematical language is an important building block for future content.

Using real-world context examples, the nature of algebraic expressions can be explored. For example, suppose the cost of cell phone service for a month is represented by the expression $0.40s + 12.95$. Students can analyze how the coefficient of 0.40 represents the cost of one minute (40¢), while the constant of 12.95 represents a fixed, monthly fee, and *s* stands for the number of cell phone minutes used in the month. Similar real-world examples, such as tax rates, can also be used to explore the meaning of expressions.

<u>Common Misconceptions from Ohio's Model Curricula:</u>
Students may believe that the use of algebraic expressions is merely the abstract manipulation of symbols. Use of real-world context examples to demonstrate the meaning of the parts of algebraic expressions is needed to counter this misconception

<u>Strategies for Diverse Learners:</u>
Problems of varying difficulty are incorporated into the problem sets.

<u>Literacy Standards Considerations:</u>

- **I can demonstrate that an exponential function has a constant multiplier over equal intervals.**

Students would need to be familiar with the definitions of key terms. Word Problems are included as a part of the practice set - students will have to interpret the key terms as they apply to the problem.

- **I can identify situations that display equal ratios of change over equal intervals and can be modeled with exponential functions.**

Students would need to be familiar with the definitions of key terms. Word Problems are included as a part of the practice set - students will have to interpret the key terms as they apply to the problem.

- **I can use graphs or tables to compare the rates of change of linear, quadratic, and exponential functions.**

Students would need to be familiar with the definitions of key terms.

- **I can rewrite exponential functions using the properties of exponents.**

Students would need to be familiar with the definitions of key terms.

- **I can interpret the parameters of an exponential function in real-life problems.**

Students would need to be familiar with the definitions of key terms. Word Problems are included as a part of the practice set - students will have to interpret the key terms as they apply to the problem.

- **I can graph exponential functions, identifying key features such as the intercepts, maximum and/or minimum values, asymptotes, and end behavior of the graph.**

Students would need to be familiar with the definitions of key terms.

- **I can construct a scatter plot, use technology to find an exponential function of best fit, and use that function to make predictions.**

Students would need to be familiar with the definitions of key terms. Word Problems are included as a part of the practice set - students will have to interpret the key terms as they apply to the problem.

Instructional Resources:

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- **I can demonstrate that an exponential function has a constant multiplier over equal intervals.**

- **I can identify situations that display equal ratios of change over equal intervals and can be modeled with exponential functions.**

- **I can use graphs or tables to compare the rates of change of linear, quadratic, and exponential functions.**

- **I can rewrite exponential functions using the properties of exponents.**
- **I can interpret the parameters of an exponential function in real-life problems.**
- **I can graph exponential functions, identifying key features such as the intercepts, maximum and/or minimum values, asymptotes, and end behavior of the graph.**
- **I can construct a scatter plot, use technology to find an exponential function of best fit, and use that function to make predictions.**

Assessment:
Formative:
- exit slips
- observation of students working on problems in class
- observation of student work from outside of class practice problems

Summative:
- unit test questions

Assessment Resources:
http://www.illustrativemathematics.org/
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